



LIFECON DELIVERABLE D 2.2

Statistical Condition Management and Financial Optimisation in Lifetime Management of Structures

**Erkki Vesikari
VTT Building and Transport**

Shared-cost RTD project

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Lifecon Deliverables

Deliverable No	Title of the Deliverable
D1.1	Generic technical handbook for a predictive life cycle management system of concrete structures (Lifecon LMS)
D1.2	Generic instructions on requirements, framework and methodology for IT-based decision support tool for Lifecon LMS
D1.3	IT-based decision support tool for Lifecon LMS
D2.1	Reliability based methodology for lifetime management of structures
D2.2	Statistical condition management and financial optimisation in lifetime management of structures <ul style="list-style-type: none"> • Part 1: Markov chain based LCC analysis • Part 2: Reference structure models for prediction of degradation
D2.3	Methods for optimisation and decision making in lifetime management of structures <ul style="list-style-type: none"> • Part I: Multi attribute decision aid methodologies (MADA) • Part II: Quality function deployment (QFD) • Part III: Risk assessment and control
D3.1	Prototype of condition assessment protocol
D3.2	Probabilistic service life models for reinforced concrete structures
D4.1	Definition of decisive environmental parameters and loads
D4.2	Instructions for quantitative classification of environmental degradation loads onto structures
D4.3	GIS-based national exposure modules and national reports on quantitative environmental degradation loads for chosen objects and locations
D5.1	Qualitative and quantitative description and classification of RAMS (Reliability, Availability, Maintainability, Safety) characteristics for different categories of repair materials and systems
D5.2	Methodology and data for calculation of life cycle costs (LCC) of maintenance and repair methods and works
D5.3	Methodology and data for calculation of LCE (Life Cycle Ecology) in repair planning
D6.1	Validation of Lifecon LMS and recommendations for further development

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Markov Chain, LCC analysis, degradation models, concrete structures

Abstract

The Markov Chain is a calculation method that is especially suitable for a life cycle management system. It makes it possible to manage the condition of structures on a statistical basis. When combined with a LCC analysis it enables the upkeep of structures in a financially optimised way. The environmental impacts and user costs can also be included in the optimisation. Thus it serves as an ideal mathematical basis for a predictive, probabilistic, integrated and LCC analysis based management system for structures.

In the first part of this Deliverable the Markov Chain based LCC analysis method is introduced. The idea of the Markov Chain based LCC analysis is to reproduce the condition of the structure during the whole design period taking into account both degradation and the effects of maintenance, repair and rehabilitation (MR&R) actions so that the condition requirements are fulfilled and to calculate the MR&R costs accordingly during that design period.

The whole life cycle action profile during the given time frame is reproduced as a series of annual condition state distributions. The condition state distributions are predicted using transition probability matrices for both degradation and the effects of MR&R actions. As a result the probability that the structure is at any condition state at any year during the design period can be predicted.

There are two ways for the definition and timing of MR&R actions: automatic and manual. In the automatic version of the LCC analysis the definition and timing of MR&R actions are performed automatically using the decision tree method. An automatic LCC analysis guards self-actingly that the pre-set condition criteria are never exceeded and defines the timings of actions accordingly. In a manual analysis the definition and timing of MR&R actions is done manually. In principle a designer can specify whatever actions to be applied at any time during the design period. However, even in the manual analysis method many automatic features are available.

In the second part of this Deliverable the Reference structure models for prediction of degradation are introduced. The reference structure degradation models are of two types: 1) surface damage and 2) crack damage. As programmed in Excel worksheets they are interactive, i.e. they immediately respond to the input data given by the user. The input data comprise of parameters for environmental burdens, structural details and material properties.

There were two bases for the development of the Reference structure models: 1) computer simulation of degradation and 2) observations of degradation on the network of reference bridges. The degradation models can be utilised in the Markov Chain based LCC analyses at both the object level and network level management systems and in service life design of concrete structures.

The full equations of the Reference structure models are presented in the Appendix.

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PART 1: MARKOV CHAIN BASED LCC ANALYSIS

1 Properties of the Markov Chain Method

1.1 Introduction

The Markov Chain is a calculation method that is especially suitable for a life cycle management system. It makes it possible to manage the condition of structures on a statistical basis. When combined with a LCC analysis it enables the upkeep of structures in a financially optimised way. The environmental impacts and user costs can also be included in the optimisation. Thus it serves as an ideal mathematical basis for a predictive, probabilistic, integrated and LCC analysis based management system for structures.

Instead of continuous functions of mean and standard deviation for the condition of structures these mathematics uses a condition state distribution according to a discrete set of predefined condition states. At the network level the percentages of structures at each condition state define the condition distribution vector of structures. At the object level the condition distribution vector can be interpreted as expressing the probability of a single structure to be at any of the condition states.

The Markov Chain method makes it possible to reproduce the performance of structures as a series of condition state distributions over the treated time frame. At an object level LCC analysis the Markov Chain method is applied for evaluating the condition related effects of both degradation and actions of maintenance, repair and rehabilitation (MR&R). The actions are automatically triggered whenever the maximum allowable probability for the defined limit state is exceeded. The following advantages of the Markov Chain method are noted:

- treatment of condition-related effects in a fully probabilistic manner,
- capability of describing parallel time dependent processes such as the protection of structures by coatings which are under the process of deterioration,
- capability of straightforward combining sequential degradation processes such as the process of depassivation by carbonation or chloride contamination and active corrosion of reinforcement,
- probabilistic triggering of actions,
- possibility of defining the volume (area) of actions based on the condition related criteria (patching),
- possibility of calculating risk costs as the product of "failure costs" and the "failure probability".

One of the most important properties of the Markov Chain is that it produces a mathematically linear system allowing linear optimisation. This property can be exploited in the optimisation of MR&R resources and methods. Because of these capabilities the Markov Chain has been the most commonly used mathematics in the existing predictive facility management systems in the world.

1.2 Reproduction of Performance over Time by the Markov Chain Method

In the Markov Chain method the number of states is not restricted, but the calculations are easier if the number of states is not too large. As an example we may use a 5-state system the states corresponding to the degrees of damage 0, 1, 2, 3, 4. The condition state 0 represents the best and 4 the poorest condition as presented in Table 1. The condition state 3 defines the maximum allowable degradation, i.e. the limit state of degradation.

Table 1. Interpretation of condition states (example).

Condition state (Degree of damage)	State definition
0	Initial state, no degradation
1	1/3 limit state
2	2/3 limit state
3	limit state
4	post limit state

The condition state vector W expresses the fraction of structures, w_i , in each state i . The fraction of structures may in principle be determined on volumetric, areal or longitudinal bases i.e. the common functional unit should be used. As all structures must be at any of the condition states the sum of w_i 's must be 1. A sample of the condition state vector is given in Figure 1.

State	0	1	2	3	4
Condition state vector W	w_0	w_1	w_2	w_3	w_4

Figure 1. Condition state vector.

The changes in a condition state vector with time are predicted by transition probability matrices. There are two kinds of transition probability matrices: (1) matrices for degradation and (2) matrices for action effects. To predict only the effects of degradation on the condition of structures the matrices for degradation are used. The matrices for action effects are used only when MR&R actions are performed.

The general form of a transition probability matrix for degradation is presented in Figure 2. Each matrix element p_{mn} expresses the probability that a structure being initially at the state m will be dropped to the state n during one year. As it is impossible that the condition state of a structure is improved without repair actions the elements in a degradation matrix below the diagonal line are 0. The sum of elements in each row of a transition probability matrix must be 1. At the lower right corner of the matrix the value of the probability element is always 1 as the structures in the highest possible condition state always stay at the same condition state.

State	0	1	2	3	4
0	p ₀₀	p ₀₁	p ₀₂	p ₀₃	p ₀₄
1	0	p ₁₁	p ₁₂	p ₁₃	p ₁₄
2	0	0	p ₂₂	p ₂₃	p ₂₄
3	0	0	0	p ₃₃	p ₃₄
4	0	0	0	0	1

Figure 2. Transition probability matrix for degradation.

Changes after n years are predicted by multiplying the initial condition state vector, $W(0)$, by the transition matrix n times in row, as shown in Figure 3. In the example of Figure 3 we have assumed that all structures start off in perfect condition. So the initial condition state distribution is $| 1, 0, 0, 0, 0 |$. We have also assumed that during one year the structures either remain in the same condition state where they were at the beginning of that year or drop to the next state (dropping more than 1 state in a year is excluded as this usually does not happen in practice).

The condition state distribution for each year, $W(t)$, is obtained by multiplying the condition state distribution of the previous year by the transition matrix P, as given by Equation 1. The result is shown graphically in Figure 4.

$$W(t) = W(t-1) \times P \quad (1)$$

Transition probability matrix P					
State	0	1	2	3	4
0	0.61	0.39	0	0	0
1	0	0.74	0.26	0	0
2	0	0	0.82	0.18	0
3	0	0	0	0.91	0.09
4	0	0	0	0	1

Condition state distribution W(t)						
Year	State					Average DoD
	0	1	2	3	4	
0	1.000	0.000	0.000	0.000	0.000	0.00
1	0.610	0.390	0.000	0.000	0.000	0.39
2	0.372	0.527	0.101	0.000	0.000	0.73
3	0.227	0.535	0.220	0.018	0.000	1.03
4	0.138	0.484	0.319	0.056	0.002	1.30
5	0.084	0.412	0.388	0.109	0.007	1.54
6	0.052	0.338	0.425	0.169	0.016	1.76
7	0.031	0.270	0.437	0.230	0.032	1.96
8	0.019	0.212	0.428	0.288	0.052	2.14
9	0.012	0.165	0.406	0.339	0.078	2.31
10	0.007	0.126	0.376	0.382	0.109	2.46
11	0.004	0.096	0.341	0.415	0.143	2.60
12	0.003	0.073	0.305	0.439	0.181	2.72
13	0.002	0.055	0.269	0.454	0.220	2.84
14	0.001	0.041	0.235	0.462	0.261	2.94
15	0.001	0.031	0.203	0.463	0.303	3.04
16	0.000	0.023	0.175	0.458	0.344	3.12
17	0.000	0.017	0.149	0.448	0.385	3.20
18	0.000	0.013	0.127	0.434	0.426	3.27
19	0.000	0.010	0.107	0.418	0.465	3.34
20	0.000	0.007	0.091	0.400	0.502	3.40
21	0.000	0.005	0.076	0.380	0.538	3.45
22	0.000	0.004	0.064	0.360	0.573	3.50
23	0.000	0.003	0.053	0.339	0.605	3.55
24	0.000	0.002	0.044	0.318	0.635	3.59
25	0.000	0.002	0.037	0.297	0.664	3.62
26	0.000	0.001	0.031	0.277	0.691	3.66
27	0.000	0.001	0.026	0.258	0.716	3.69
28	0.000	0.001	0.021	0.239	0.739	3.72
29	0.000	0.000	0.018	0.221	0.760	3.74
30	0.000	0.000	0.015	0.205	0.780	3.77
31	0.000	0.000	0.012	0.189	0.799	3.79
32	0.000	0.000	0.010	0.174	0.816	3.81
33	0.000	0.000	0.008	0.160	0.832	3.82
34	0.000	0.000	0.007	0.147	0.846	3.84
35	0.000	0.000	0.006	0.135	0.859	3.85
36	0.000	0.000	0.005	0.124	0.871	3.87
37	0.000	0.000	0.004	0.114	0.883	3.88
38	0.000	0.000	0.003	0.104	0.893	3.89
39	0.000	0.000	0.003	0.095	0.902	3.90
40	0.000	0.000	0.002	0.087	0.911	3.91
41	0.000	0.000	0.002	0.080	0.919	3.92
42	0.000	0.000	0.001	0.073	0.926	3.92
43	0.000	0.000	0.001	0.067	0.932	3.93
44	0.000	0.000	0.001	0.061	0.938	3.94
45	0.000	0.000	0.001	0.055	0.944	3.94
46	0.000	0.000	0.001	0.051	0.949	3.95
47	0.000	0.000	0.001	0.046	0.953	3.95
48	0.000	0.000	0.000	0.042	0.957	3.96
49	0.000	0.000	0.000	0.038	0.961	3.96
50	0.000	0.000	0.000	0.035	0.965	3.96

Figure 3. Calculation principle of the Markov Chain method.

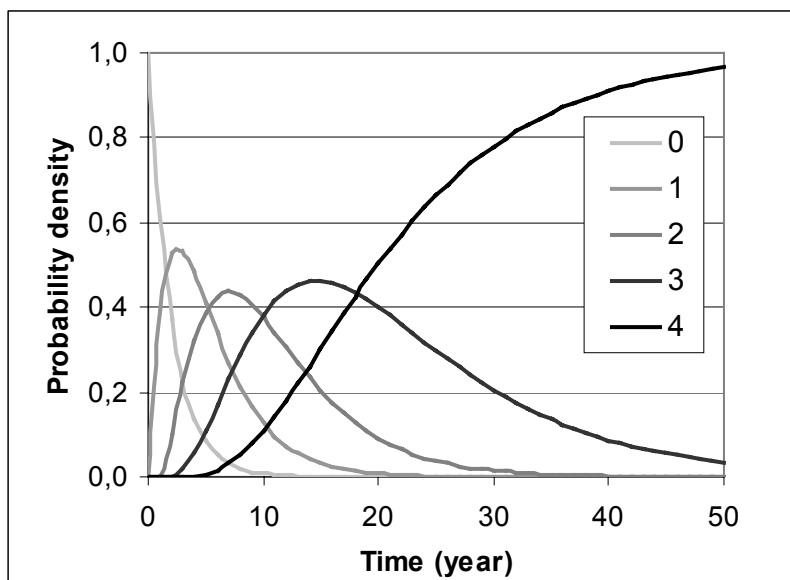


Figure 4. Probability density functions for condition states 0 - 4 calculated by the Markov Chain method.

The concept of degree of degradation, DoD, is used as equivalent for condition state. The average DoD at each year, $E(t)$, is obtained by multiplying the scale vector, $R = \{0, 1, 2, 3, 4\}$, by the condition state distribution (vector multiplication), as seen in Equation 2. The result is shown graphically in Figure 5.

$$E(t) = W(t) \times R \quad (2)$$

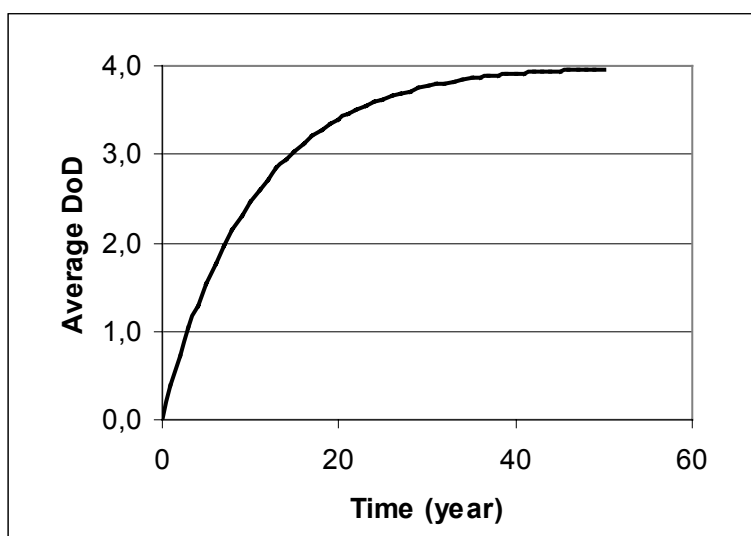


Figure 5. Average degree of degradation (=condition state) as a function of time calculated by Markov Chain.

By the Markov Chain process the probability of structures being at any of the condition states can be predicted. Figure 4 shows the probability density functions for each condition state 0 - 4.

The probability density function of condition state 4 is also a cumulative probability function as there is no bigger condition state defined. So all structures that drop to condition state 4 also remain at that state. The cumulative probability curve for condition state 3 is obtained by adding the probability density of state 3 to the cumulative probability of state 4. Likewise the cumulative probability curve for state 2 is obtained by adding the probability density of state 2 to the cumulative probability of state 3, etc.

Using the numerical data of the earlier example the cumulative distribution curves presented in Figure 6 can be obtained.

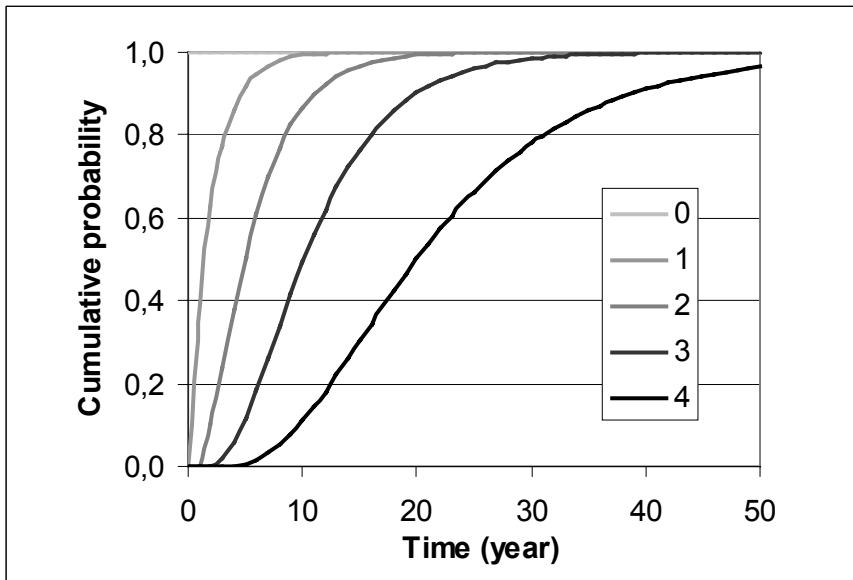


Figure 6. Cumulative probability functions for the condition to exceed or to be equal to condition states 0 - 4 determined by the Markov Chain method.

If the condition state 3 is the limit state for service life the probability of exceeding the service life can be read from the curve for condition state 3 in Figure 6. So the cumulative curves for the limit state can be interpreted as the cumulative function of service life.

1.3 Action Effects in a Markov Chain Process

In a Markov Chain process the effects of MR&R actions can be taken into account by transition probability matrices of action effects. The appearance of an action effect matrix is as follows:

State	0	1	2	3	4
0	p_{00}	0	0	0	0
1	p_{10}	p_{11}	0	0	0
2	p_{20}	p_{21}	p_{22}	0	0
3	p_{30}	p_{31}	p_{32}	p_{33}	0
4	p_{40}	p_{41}	p_{42}	p_{43}	p_{44}

Figure 7. Transition probability matrix for MR&R action effects.

As it is assumed that the condition state of a structure is always improved or at least remains the same as a result of an action, all the probability elements above the diagonal are 0. Other elements may have a value from 0 to 1. However heavy repair actions normally bring the structures close to the perfect condition so that the elements in the first column of the matrix are near 1.

A typical action effect matrix can be seen at the top of Figure 8. The calculation table is programmed so that a repair is done every time when signed by 1 in the column at the left-side of the figure. The action effects can be readily seen in the condition state distributions and the average DoD curve presented in Figure 9.

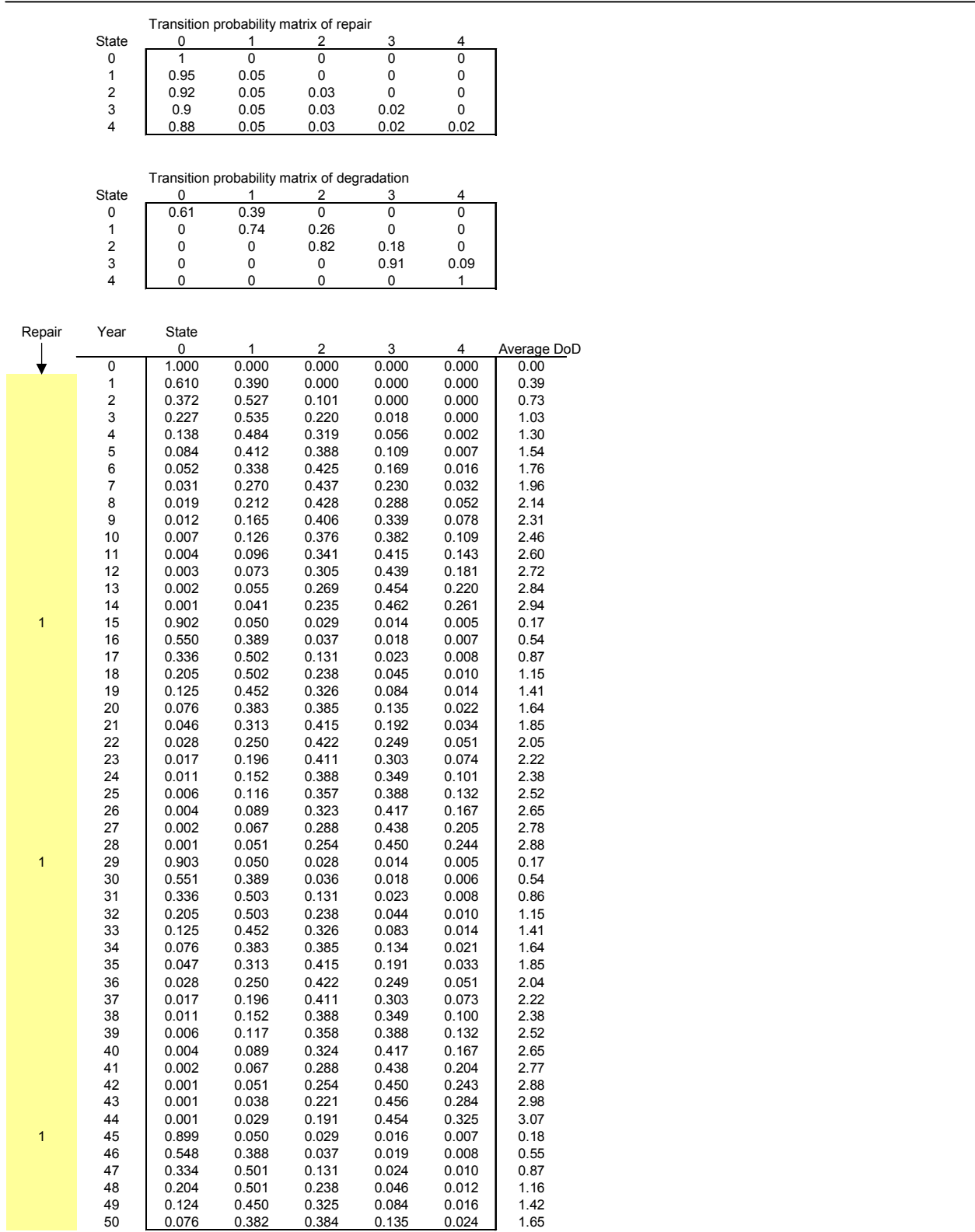


Figure 8. Prediction of condition with Markov Chain degradation and action effect matrices.

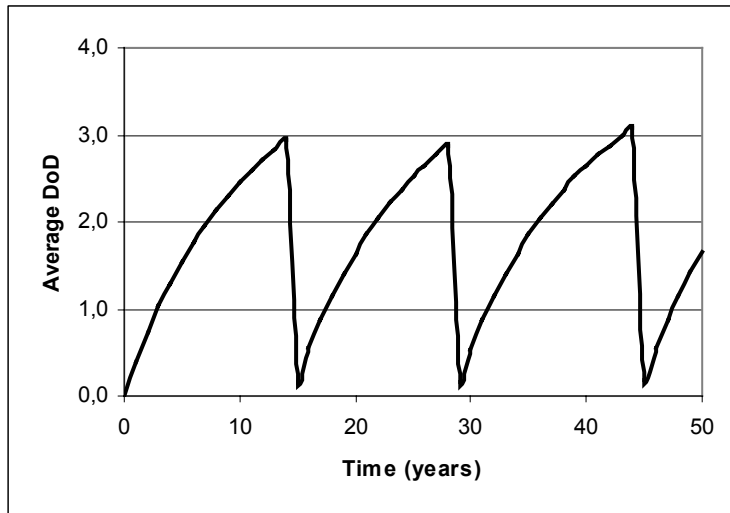


Figure 9. The average DoD with time showing the effects of repair on the condition of a structure.

Every time when it is decided to do a MR&R action, the action effect matrix is applied to shift the condition state of the structure to the predicted condition state distribution after the repair. The rate of degradation may also be changed after the repair (as compared to the original). So the matrix of degradation is changed accordingly, taking into account the material and structural properties of the repaired structure.

In the case of coatings the condition state of the structure is not considered to be changed but the rate of further degradation is reduced. So no action effect matrix is applied in the case of coatings but the degradation matrix is changed to exhibit a slower degradation rate.

2 Process of Producing Transition Probability Models

Figures 2 - 9 showed the basic idea of condition prediction by the Markov Chain method. Knowing the initial condition state distribution and the transition probability matrices for degradation and action effects the prediction over time can be done simply by repeated matrix multiplications.

The practical problem in the condition prediction of structures is of course how to determine the transition probability matrices for degradation and action effects. The available methods to produce these models are dealt with in this chapter.

2.1 Degradation Models

2.1.1 Assumptions

As an assumption we have defined the condition states with gradually increasing phases of degradation (ref. Table 1). In principle the degradation may occur as increasing depth of penetration or internal damage in the material. Accordingly we assume that the degradation function which shows the increasing damage as a function of time can be expressed in the following way

$$DoD(t) = N_{LS} \cdot \frac{s(t)}{s_{max}} = N_{LS} \cdot f(t) \quad (3)$$

where

- DoD is scaled degradation function (degree of damage as a function of time),
- N_{LS} limit state corresponding to the maximum accepted depth of penetration or amount of damage in the scale 0, 1, 2, 3 ...N,
- $s(t)$ depth of penetration or amount of damage as a function of time,
- s_{max} maximum depth of penetration or amount of damage corresponding to the limit state N_{LS} , and
- $f(t)$ unscaled degradation function (value between 0 and 1).

In the following examples the unscaled degradation function is assumed to be of the following form (ref. Part 2 "Reference Structure Models for Prediction of Degradation").

$$f = a \cdot t^n \quad (4)$$

where

- a is coefficient, and
- n exponent of time.

According to Equations 3 and 4 we can write an expression for the degradation function as follows:

$$DoD = c \cdot t^n \quad (5)$$

where

c is coefficient, and
n exponent.

The relationship between the coefficients c and a is:

$$c = N_{LS} \cdot a \quad (6)$$

The exponent n is the same in both equations.

In the following examples we have assumed that there is a 5-state rating system from 0 to 4 and the limit state is 3 ($N_{LS}=3$) according to Table 1.

For the transition probabilities of the degradation models the following assumptions are made (ref. Figure 2):

1. The condition of structures cannot be improved during the process.
2. The condition state can either remain the same or shift to the next state within one year (transition period).

With the first assumption the so-called "self healing" is not possible. Because of this assumption all transition probabilities below the diagonal probabilities are zero.

The second assumption can be considered reasonable and moderate for normal deterioration processes of concrete structures in outdoor conditions. Due to this assumption, all probabilities above those next to diagonal ones are zero.

As the structures must either remain at the same state or drop to the next one within a year, the sum of the probabilities of remaining (diagonal elements) and dropping to the next state (elements next to the diagonal ones) must be 1. Thus the "remain-in-state" probabilities can be calculated by subtracting the "drop-from-state" probabilities from 1.

Since there is no higher condition state than 4, structures at that condition state must always remain at that condition state, and so the corresponding transition probability, p_{44} , is always 1 (at the lower right corner of the matrix).

In a 5-state system the assumed transition probability matrix would look like Figure 10.

P =	<table style="border-collapse: collapse; width: 100%; text-align: center;"> <tr> <td style="padding: 5px;">1-p₀₁</td> <td style="padding: 5px;">p₀₁</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1-p₁₂</td> <td style="padding: 5px;">p₁₂</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1-p₂₃</td> <td style="padding: 5px;">p₂₃</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1-p₃₄</td> <td style="padding: 5px;">p₃₄</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> </tr> </table>	1-p ₀₁	p ₀₁	0	0	0	0	1-p ₁₂	p ₁₂	0	0	0	0	1-p ₂₃	p ₂₃	0	0	0	0	1-p ₃₄	p ₃₄	0	0	0	0	1
1-p ₀₁	p ₀₁	0	0	0																						
0	1-p ₁₂	p ₁₂	0	0																						
0	0	1-p ₂₃	p ₂₃	0																						
0	0	0	1-p ₃₄	p ₃₄																						
0	0	0	0	1																						

Figure 10. Assumed transition probability matrix for degradation.

The "drop-from-state" probabilities are the only unknown parameters to produce a transition probability matrix for degradation. Almost all kinds of real degradation data can be used as the reference to solve the unknown transition probabilities. The most usual ways of presenting the reference data are the following:

1. known average degradation curve
2. known distribution of service life.

The unknown transition probabilities can be solved in principle by the "Minimum deviation method". In practical calculations Excel Solver can be used. The method is described in detail in the following chapters.

2.1.2 Minimum Deviation Method with Known Average Degradation Curve

The principle in the Minimum deviation method is to minimise the difference between the Markov Chain average and the reference curves within a selected time frame by changing the values of the "drop-from-state" transition probabilities. As this difference calculated for each year might be positive or negative the square of the difference is determined. The sum of the squares of differences is minimised as an optimisation problem:

Minimise

$$SumDev^2 = \sum_{j=1}^t (E(j) - DoD(j))^2 \quad (7)$$

by changing the values of $p_{i;i+1}$ for each i ($0 \leq p_{i;i+1} \leq 1$).

where

SumDev² is the sum of annual squares of deviation (object function to be minimised),

E(j) average degradation curve determined by the Markov Chain method,

DoD(j) degradation according to the reference degradation curve,

j year, and

i state.

Considering the coefficient c and the exponent of time n are known for $E(j)$, the corresponding values for probabilities $p_{i;i+1}$ can be solved as presented in Figure 11.

Reference degradation curve: $DoD=(Coefficient)^{t^{(Exponent)}}$

Transition probability matrix									
State	0	1	2	3	4	P_{ii}	$P_{i,i+1}$		
0	0.217	0.783	0	0	0	0.217	0.783		
1	0	0.839	0.161	0	0	0.839	0.161		
2	0	0	0.942	0.058	0	0.942	0.058		
3	0	0	0	0.941	0.059	0.941	0.059		
4	0	0	0	0	1	1			
						SumDev^2:	0.001		
						Coefficient	0.8		
						Exponent	0.4		
Year	State	0	1	2	3	4	Markov	Ref	Dev^2
0	0	1.000	0.000	0.000	0.000	0.000	0.00	0.00	0.000
1	0	0.217	0.783	0.000	0.000	0.000	0.78	0.80	0.000
2	0	0.047	0.827	0.126	0.000	0.000	1.08	1.06	0.001
3	0	0.010	0.730	0.252	0.007	0.000	1.26	1.24	0.000
4	0	0.002	0.621	0.355	0.022	0.000	1.40	1.39	0.000
5	0	0.000	0.522	0.434	0.041	0.002	1.52	1.52	0.000
6	0	0.000	0.438	0.493	0.064	0.004	1.63	1.64	0.000
7	0	0.000	0.368	0.535	0.089	0.008	1.74	1.74	0.000
8	0	0.000	0.309	0.563	0.115	0.013	1.83	1.84	0.000
9	0	0.000	0.259	0.580	0.141	0.020	1.92	1.93	0.000
10	0	0.000	0.217	0.588	0.167	0.028	2.01	2.01	0.000
11	0	0.000	0.182	0.589	0.191	0.038	2.09	2.09	0.000
12	0	0.000	0.153	0.584	0.214	0.049	2.16	2.16	0.000
13	0	0.000	0.128	0.574	0.236	0.062	2.23	2.23	0.000
14	0	0.000	0.107	0.561	0.255	0.076	2.30	2.30	0.000
15	0	0.000	0.090	0.546	0.273	0.091	2.36	2.36	0.000
16	0	0.000	0.076	0.529	0.289	0.107	2.43	2.43	0.000
17	0	0.000	0.063	0.510	0.303	0.124	2.49	2.48	0.000
18	0	0.000	0.053	0.490	0.314	0.142	2.55	2.54	0.000
19	0	0.000	0.045	0.470	0.325	0.161	2.60	2.60	0.000
20	0	0.000	0.037	0.450	0.333	0.180	2.65	2.65	0.000
21	0	0.000	0.031	0.430	0.339	0.199	2.71	2.70	0.000
22	0	0.000	0.026	0.410	0.345	0.219	2.76	2.75	0.000
23	0	0.000	0.022	0.390	0.348	0.240	2.81	2.80	0.000
24	0	0.000	0.019	0.371	0.350	0.260	2.85	2.85	0.000
25	0	0.000	0.016	0.352	0.351	0.281	2.90	2.90	0.000
26	0	0.000	0.013	0.334	0.351	0.302	2.94	2.94	0.000
27	0	0.000	0.011	0.317	0.350	0.322	2.98	2.99	0.000
28	0	0.000	0.009	0.300	0.348	0.343	3.02	3.03	0.000
29	0	0.000	0.008	0.284	0.345	0.363	3.06	3.08	0.000
30	0	0.000	0.006	0.269	0.341	0.384	3.10	3.12	0.000
31	0	0.000	0.005	0.254	0.337	0.404	3.14	3.16	0.000
32	0	0.000	0.005	0.240	0.332	0.424	3.17	3.20	0.000
33	0	0.000	0.004	0.227	0.326	0.443	3.21	3.24	0.000
34	0	0.000	0.003	0.214	0.320	0.463	3.24	3.28	0.000
35	0	0.000	0.003	0.202	0.314	0.481	3.27	3.32	0.000
36	0	0.000	0.002	0.191	0.307	0.500	3.30	3.35	0.000
37	0	0.000	0.002	0.180	0.300	0.518	3.33	3.39	0.000
38	0	0.000	0.002	0.170	0.293	0.536	3.36	3.43	0.000
39	0	0.000	0.001	0.160	0.285	0.553	3.39	3.46	0.000
40	0	0.000	0.001	0.151	0.278	0.570	3.42	3.50	0.000
41	0	0.000	0.001	0.142	0.270	0.586	3.44	3.53	0.000
42	0	0.000	0.001	0.134	0.263	0.602	3.47	3.57	0.000
43	0	0.000	0.001	0.127	0.255	0.618	3.49	3.60	0.000
44	0	0.000	0.001	0.119	0.247	0.633	3.51	3.63	0.000
45	0	0.000	0.000	0.112	0.240	0.647	3.53	3.67	0.000
46	0	0.000	0.000	0.106	0.232	0.662	3.55	3.70	0.000
47	0	0.000	0.000	0.100	0.225	0.675	3.57	3.73	0.000
48	0	0.000	0.000	0.094	0.217	0.688	3.59	3.76	0.000
49	0	0.000	0.000	0.089	0.210	0.701	3.61	3.79	0.000
50	0	0.000	0.000	0.083	0.203	0.714	3.63	3.83	0.000

Figure 11. Solution of the transition probabilities when the mean degradation curve is known.

In this example it was assumed that the coefficient c is 0.8 and n is 0.4. The corresponding annual values for the degradation curve are presented in the column "Ref". The annual values of the Markov Chain mean curve are presented in column "Markov" (ref. Chapter 1.2). The column Dev^2 shows the annual squares of the deviation between the Markov Chain curve and the reference mean curve. A condition has been set, however, that if the value of the reference curve is greater than 3 then the value of Dev^2 is 0.

In Figure 12 we can see the result as a graphic presentation.

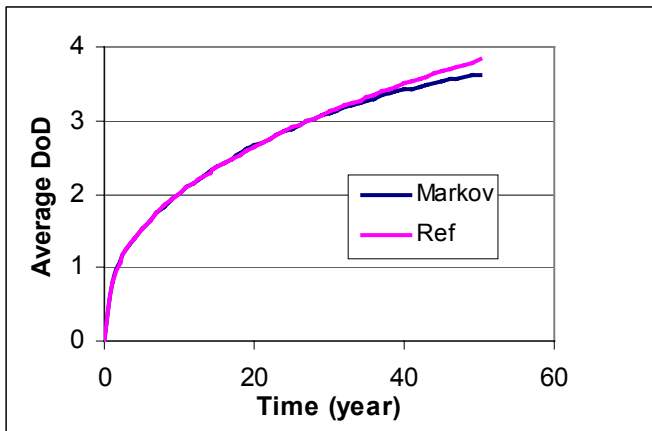


Figure 12. Comparison of the Markov Chain and the reference mean curves.

The conformity of the Markov Chain and the reference mean curve is very good when DoD is between 0 and 3. If DoD is greater than 3 the conformity is not very good as these values were not considered in the optimisation.

The above presented process is applicable when the degradation curve is linear or decelerating ($n \leq 1$). If the degradation curve is strongly accelerating ($n > 1.4$) a good conformity is not possible with constant transition probabilities.

The reason for a strongly accelerating degradation process may be that there is an "incubation time" of degradation. It means that the probability p_{01} is not constant but changes with time. Assuming that the transition probability p_{01} increases linearly from 0 to 1 we can define this probability mathematically as follows:

$$\begin{aligned} p_{01} &= \text{Min}(1; t \cdot \Delta p) \\ p_{00} &= 1 - p_{01} \end{aligned} \quad (8)$$

where

Δp is increase of the "drop-from-state" probability p_{01} in a year and
 t year from the beginning.

Consider for instance that the coefficient c is 0.0001 and the exponent of time is 2 (ref. Formula 5). Then the solution can be found as presented in Fig 13. The unknown parameter related to p_{01} is Δp .

Reference degradation curve: $DoD=(Coefficient)*t^{(Exponent)}$

Transition probability p_{01} : $p_{01}=k*t$

States:

0	$1-k*t$	$k*t$	0	0	0	k	0.000129
1	0	0.968	0.032	0	0	p_{12}	0.032
2	0	0	0.945	0.055	0	p_{23}	0.055
3	0	0	0	0.901	0.099	p_{34}	0.099
4	0	0	0	0	1		

Year	k	p_{11}	p_{22}	p_{33}	p_{44}	SumDev^2:		
	0.000129	0.968	0.945	0.901	1.000	Coefficient	0.0001	3.210
	State					Exponent	2	
	0	1	2	3	4	Average DoI	Ref	Dev^2
0	1.000	0.000	0.000	0.000	0.000	0.00	0.00	0.000
1	1.000	0.000	0.000	0.000	0.000	0.00	0.00	0.000
2	1.000	0.000	0.000	0.000	0.000	0.00	0.00	0.000
3	0.999	0.001	0.000	0.000	0.000	0.00	0.00	0.000
4	0.999	0.001	0.000	0.000	0.000	0.00	0.00	0.000
5	0.998	0.002	0.000	0.000	0.000	0.00	0.00	0.000
6	0.997	0.003	0.000	0.000	0.000	0.00	0.00	0.000
7	0.996	0.003	0.000	0.000	0.000	0.00	0.00	0.000
8	0.995	0.004	0.000	0.000	0.000	0.00	0.01	0.000
9	0.994	0.005	0.000	0.000	0.000	0.01	0.01	0.000
10	0.993	0.006	0.001	0.000	0.000	0.01	0.01	0.000
11	0.992	0.008	0.001	0.000	0.000	0.01	0.01	0.000
12	0.990	0.009	0.001	0.000	0.000	0.01	0.01	0.000
13	0.988	0.010	0.001	0.000	0.000	0.01	0.02	0.000
14	0.987	0.012	0.001	0.000	0.000	0.02	0.02	0.000
15	0.985	0.013	0.002	0.000	0.000	0.02	0.02	0.000
16	0.983	0.015	0.002	0.000	0.000	0.02	0.03	0.000
17	0.980	0.017	0.002	0.000	0.000	0.02	0.03	0.000
18	0.978	0.018	0.003	0.001	0.000	0.03	0.03	0.000
19	0.976	0.020	0.003	0.001	0.000	0.03	0.04	0.000
20	0.973	0.022	0.004	0.001	0.000	0.03	0.04	0.000
21	0.971	0.024	0.004	0.001	0.000	0.04	0.04	0.000
22	0.968	0.026	0.005	0.001	0.000	0.04	0.05	0.000
23	0.965	0.028	0.005	0.001	0.001	0.04	0.05	0.000
24	0.962	0.030	0.006	0.001	0.001	0.05	0.06	0.000
25	0.959	0.032	0.007	0.002	0.001	0.05	0.06	0.000
26	0.956	0.034	0.007	0.002	0.001	0.06	0.07	0.000
27	0.952	0.037	0.008	0.002	0.001	0.06	0.07	0.000
28	0.949	0.039	0.009	0.002	0.001	0.07	0.08	0.000
29	0.945	0.041	0.009	0.002	0.002	0.07	0.08	0.000
30	0.942	0.044	0.010	0.003	0.002	0.08	0.09	0.000
31	0.938	0.046	0.011	0.003	0.002	0.09	0.10	0.000
32	0.934	0.048	0.012	0.003	0.002	0.09	0.10	0.000
33	0.930	0.051	0.013	0.004	0.003	0.10	0.11	0.000
34	0.926	0.053	0.014	0.004	0.003	0.10	0.12	0.000
35	0.922	0.056	0.015	0.004	0.003	0.11	0.12	0.000
36	0.918	0.058	0.016	0.005	0.004	0.12	0.13	0.000
37	0.913	0.061	0.017	0.005	0.004	0.13	0.14	0.000
38	0.909	0.063	0.018	0.006	0.005	0.13	0.14	0.000
39	0.904	0.066	0.019	0.006	0.005	0.14	0.15	0.000
40	0.899	0.068	0.020	0.006	0.006	0.15	0.16	0.000
41	0.895	0.071	0.021	0.007	0.007	0.16	0.17	0.000
42	0.890	0.073	0.022	0.007	0.007	0.17	0.18	0.000
43	0.885	0.076	0.023	0.008	0.008	0.18	0.18	0.000
44	0.880	0.079	0.024	0.008	0.009	0.19	0.19	0.000
45	0.875	0.081	0.026	0.009	0.010	0.20	0.20	0.000
46	0.870	0.084	0.027	0.009	0.011	0.21	0.21	0.000
47	0.864	0.086	0.028	0.010	0.011	0.22	0.22	0.000
48	0.859	0.089	0.029	0.010	0.012	0.23	0.23	0.000
49	0.854	0.092	0.030	0.011	0.014	0.24	0.24	0.000
50	0.848	0.094	0.032	0.012	0.015	0.25	0.25	0.000

Figure 13. Solution of transition probabilities with an "incubation time".

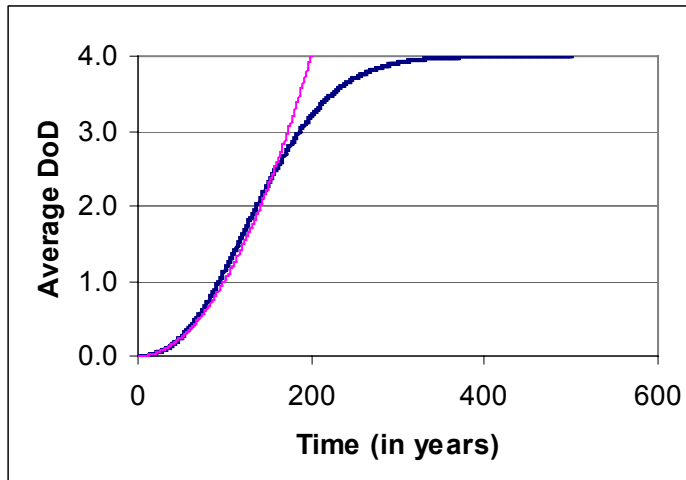


Figure 14. Comparison of the Markov Chain curve and the reference mean curve.

As seen in Figure 14, the reference degradation curve is valid only up to state 3. Thereafter the Markov Chain curve is bound to bend horizontal as there is no greater condition state than 4. So the average degradation can never be greater than 4. Because of the forced bending of the Markov Chain curve to a horizontal direction there must always be an extra state above the limit state. Without this extra state it would be impossible to have a good fitting with the reference curve.

2.1.3 Minimum Deviation Method with Known Probability Function of Service Life

Generally the probability function of service life can be defined by the following parameters:

1. Mean, μ ,
2. Coefficient of variance ($v = \sigma/\mu$), and
3. Form of distribution.

In the following example we assume that the probability degradation curve obeys the form of lognormal distribution. The limit state of degradation (which defines the end of the service life) is the condition state 3. The principle now is to compare the known distribution of service life with the cumulative Markov Chain distribution for state 3. The solution in this example is given in Figure 15.

Service life (Limit State DoD 3): 200
 Coefficient of variance: 0.7
 Assumed form of distribution: Log-normal SumDev²:
0.0549

p_{ii}	$p_{i,i+1}$
0.991	0.009
0.976	0.024
0.976	0.024
0.910	0.090

Lognorm-distribution
 Mean 200 years
 CoVar 0.7
 StDev 140 years
 S2 0.399
 M 5.099
 S 0.631

Year	State	0	1	2	Markov 3	4	Ref	Dev ²
0	1.000	0.000	0.000	0.000	0.000	0.000	0.0000	
1	1.000	0.009	0.000	0.000	0.000	0.000	0.0000	
2	1.000	0.018	0.000	0.000	0.000	0.000	0.0000	
3	1.000	0.026	0.001	0.000	0.000	0.000	0.0000	
4	1.000	0.035	0.001	0.000	0.000	0.000	0.0000	
5	1.000	0.043	0.002	0.000	0.000	0.000	0.0000	
6	1.000	0.052	0.003	0.000	0.000	0.000	0.0000	
7	1.000	0.060	0.004	0.000	0.000	0.000	0.0000	
8	1.000	0.068	0.006	0.000	0.000	0.000	0.0000	
9	1.000	0.076	0.007	0.000	0.000	0.000	0.0000	
10	1.000	0.085	0.009	0.001	0.000	0.000	0.0000	
11	1.000	0.093	0.011	0.001	0.000	0.000	0.0000	
12	1.000	0.101	0.013	0.001	0.000	0.000	0.0000	
13	1.000	0.108	0.015	0.001	0.000	0.000	0.0000	
14	1.000	0.116	0.017	0.002	0.000	0.000	0.0000	
15	1.000	0.124	0.020	0.002	0.000	0.000	0.0000	
16	1.000	0.132	0.022	0.002	0.001	0.000	0.0000	
17	1.000	0.139	0.025	0.003	0.001	0.000	0.0000	
18	1.000	0.147	0.028	0.003	0.001	0.000	0.0000	
19	1.000	0.155	0.031	0.004	0.001	0.000	0.0000	
20	1.000	0.162	0.034	0.005	0.001	0.000	0.0000	
21	1.000	0.169	0.037	0.005	0.002	0.001	0.0000	
22	1.000	0.177	0.040	0.006	0.002	0.001	0.0000	
23	1.000	0.184	0.043	0.007	0.002	0.001	0.0000	
24	1.000	0.191	0.047	0.008	0.003	0.001	0.0000	
25	1.000	0.198	0.050	0.009	0.003	0.001	0.0001	
26	1.000	0.205	0.054	0.010	0.004	0.002	0.0001	
27	1.000	0.212	0.058	0.011	0.004	0.002	0.0001	
28	1.000	0.219	0.061	0.012	0.005	0.003	0.0001	
29	1.000	0.226	0.065	0.013	0.006	0.003	0.0001	
30	1.000	0.233	0.069	0.014	0.006	0.004	0.0001	
31	1.000	0.240	0.073	0.016	0.007	0.004	0.0001	
32	1.000	0.246	0.077	0.017	0.008	0.005	0.0002	
33	1.000	0.253	0.081	0.019	0.009	0.006	0.0002	
34	1.000	0.259	0.086	0.020	0.010	0.006	0.0002	
35	1.000	0.266	0.090	0.022	0.010	0.007	0.0002	
36	1.000	0.272	0.094	0.023	0.012	0.008	0.0002	
37	1.000	0.279	0.099	0.025	0.013	0.009	0.0003	
38	1.000	0.285	0.103	0.027	0.014	0.010	0.0003	
39	1.000	0.291	0.107	0.029	0.015	0.012	0.0003	
40	1.000	0.298	0.112	0.031	0.016	0.013	0.0003	
41	1.000	0.304	0.116	0.033	0.017	0.014	0.0003	
42	1.000	0.310	0.121	0.035	0.019	0.016	0.0004	
43	1.000	0.316	0.126	0.037	0.020	0.017	0.0004	
44	1.000	0.322	0.130	0.039	0.022	0.019	0.0004	
45	1.000	0.328	0.135	0.041	0.023	0.020	0.0004	
46	1.000	0.334	0.140	0.043	0.025	0.022	0.0005	
47	1.000	0.340	0.144	0.046	0.027	0.024	0.0005	
48	1.000	0.346	0.149	0.048	0.028	0.026	0.0005	
49	1.000	0.351	0.154	0.051	0.030	0.028	0.0005	
50	1.000	0.357	0.159	0.053	0.032	0.030	0.0005	

Figure 15. Solution for a transition probability matrix when the probability function of service life is known.

In the upper part of Figure 15 the lognormal reference curve has been defined by giving the values of mean depassivation time and the coefficient of variance. The annual values of the

reference curve have been calculated in column "Ref". Next to it the probabilities of reaching the states 0, 1, 2, 3 and 4 are determined based on the Markov Chain. The probability curve of the state 3 is the one to be compared with the reference curve. The column Dev^2 shows the annual deviation between Markov Chain curve and the reference curve.

The values of the transition probabilities $p_{i,i+1}$ ($i=0 \dots 3$) are then solved in the same way as in the previous example by minimising the object function SumDev^2 .

In Figure 16 we see a graphic presentation of the reference curve and the Markov Chain curve for the state 3 after optimisation.

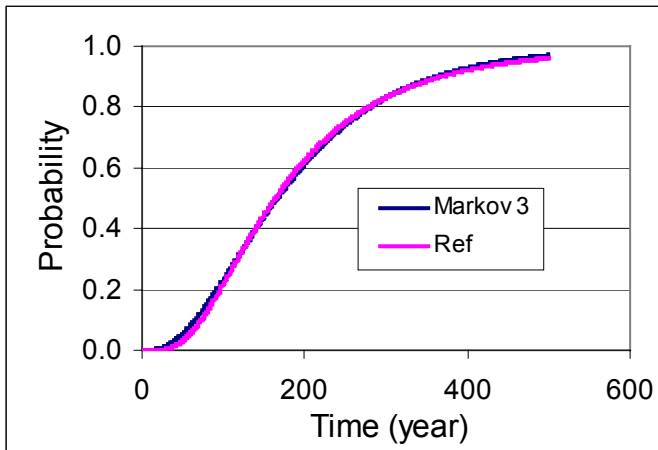


Figure 16. Comparison of the probability function for state 3 by Markov Chain and the reference probability curve of service life.

The conformity of the Markov Chain curve with the reference curve is usually good when the coefficient of variance is greater than 0.6. When $v < 0.6$ time dependent transition probabilities are needed. In that case a time dependent p_{01} as presented in the previous chapter can be used to seek a solution.

2.1.4 Automatical Conversion from a Degradation Curve to a Transition Probability Matrix

The curve fit analyses as presented above are good for problems of single structures but in analysis routines where thousands of conversions should be performed it would be far too slow. So it is important to seek a more rapid method for an automatic LCC analysis.

In the following sections a method that allows automatic conversion from a degradation curve to a transition probability matrix model is derived.

According to the rules of matrix multiplication the change in the fraction of structures in condition state i is as follows:

$$\Delta w_i = w_i^{t+1} - w_i^t = w_{i-1}^t \cdot p_{i-1,i} - w_i^t \cdot p_{i,i+1} \quad (9)$$

where

w_i is the fraction of structures in condition state i ,
 Δw_i change in the fraction of structures in condition state i ,
 $p_{i-1;i}$ transition probability from state $i-1$ to state i , and
 t year.

For the average degradation it is possible to write the following equation:

$$DoD_{avg} = 0 \cdot w_0^t + 1 \cdot w_1^t + 2 \cdot w_2^t + \dots + N \cdot w_N^t \quad (10)$$

where

DoD_{avg} is average degradation, and
 N greatest number of condition states.

So the average change in degradation (within a year) can be written as:

$$\Delta DoD = 0 \cdot \Delta w_0^t + 1 \cdot \Delta w_1^t + 2 \cdot \Delta w_2^t + \dots + (N-1) \cdot \Delta w_{N-1}^t + N \cdot \Delta w_N^t \quad (11)$$

Now when combining Equations 9 and 11 ($p_{N;N+1}=0$):

$$\begin{aligned} \Delta DoD &= 1 \cdot (w_0 \cdot p_{01} - w_1 \cdot p_{12}) + 2 \cdot (w_1 \cdot p_{12} - w_2 \cdot p_{23}) + \dots \\ &+ (N-1) \cdot (w_{N-2} \cdot p_{N-2;N-1} - w_{N-1} \cdot p_{N-1;N}) + N \cdot (w_{N-1} \cdot p_{N-1;N}) \end{aligned} \quad (12)$$

By further reduction:

$$\Delta DoD = w_0 \cdot p_{01} + w_1 \cdot p_{12} + \dots + w_{N-1} \cdot p_{N-1;N} \quad (13)$$

Equation 13 expresses the average change in degree of degradation per year.

Consider, on the other hand, that all the structures have been divided into groups according to the condition state and the change in the degradation per year is determined separately in each condition state group. Then the average change in degradation depth per year could be determined as ($\Delta DoD_{N;N+1}=0$):

$$\Delta DoD = w_0 \cdot \Delta DoD_{01} + w_1 \cdot \Delta DoD_{12} + \dots + w_{N-1} \cdot \Delta DoD_{N-1;N} \quad (14)$$

where

ΔDoD is the average change in degradation within a year,
 $\Delta DoD_{i;i+1}$ the average change of degradation per year in condition state i , and
 w_i fraction of structures in condition state i .

By comparing Formula 13 with Formula 14 it can be observed that there must be the following relationship:

$$p_{i;i+1} = \Delta DoD_{i;i+1} \quad (15)$$

The average degradation per year is the same as the rate of degradation. So instead of Equation 15 it is possible to write:

$$p_{i;i+1} = \Delta DoD_{i;i+1} = \left(\frac{\partial(DoD(t))}{\partial t} \right)_{i;i+1} \quad (16)$$

where

$\frac{\partial(DoD(t))}{\partial t}$ is the derivative of the degradation function, 1/year.

Now it is assumed that Equations 3 - 6 are valid. According to Equation 5, the following DoD function is valid:

$$DoD = N_{LS} \cdot a \cdot t^n \quad (17)$$

The derivative of this function is:

$$\frac{\partial(DoD)}{\partial t} = N_{LS} \cdot a \cdot n \cdot t^{n-1} \quad (18)$$

In the case of linear degradation (n=1) the DoD' function is:

$$\frac{\partial(DoD)}{\partial t} = N_{LS} \cdot a \quad (19)$$

So the final result becomes:

$$p_{i;i+1} = N_{LS} \cdot a \quad (20)$$

In general it can be concluded that the transition probabilities $p_{i;i+1}$ are proportional to the rate of degradation and that they can be determined as the derivative function of DoD(t) between the states $i;i+1$. As DoD is generally not constant between successive condition states, the value of DoD' must be determined as an average or intermediate value between states i and $i+1$. The result is not exact but fairly good approximations can be obtained.

The automatic conversion equations for the transition probabilities would be the following:

$$p_{i;i+1} = n \cdot N_{LS} \cdot a \cdot t_{opt;i;i+1}^{n-1} \quad (21)$$

$$t_{opt;i;i+1} \approx t_i + z_d \cdot (t_{i+1} - t_i) \quad (22)$$

$$t_i = \left(\frac{i}{N_{LS} \cdot a} \right)^n \text{ for each state } i=0 \dots N_{LS} \quad (23)$$

where

$t_{\text{opt};i;i+1}$ is optimal moment of time for determination of the "drop-from-state" transition probabilities $p_{i;i+1}$,
 i state (0, 1, 2, ..., N_{LS}),
 t_i moment of time when degradation attains state i ,
 n the exponent of time of the degradation function, and
 z_d division parameter ($0 \leq z_d \leq 1$).

A degradation curve DoD with the optimal moments of time is depicted schematically in Figure 17.

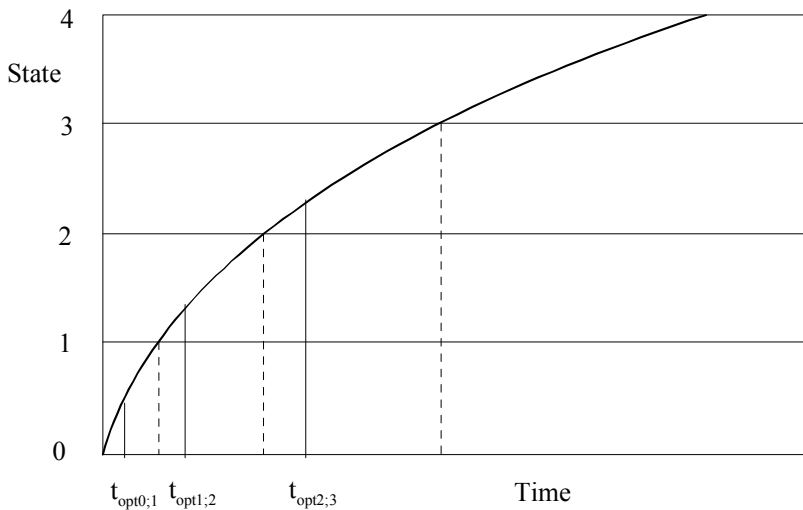


Figure 17. Degradation curve and the optimal times for state intervals $i;i+1$ (schematic presentation).

The value for the division parameter is sought experimentally. With the degradation curve of the form of Equation 17 and the exponent $n = 0.4$ the division parameter z_d is approximately $1/6$. Respectively with the exponent $n = 1.4$ the division parameter is approximately $1/2$. The intermediate values can be interpolated.

By the above-presented guidelines the automatic conversion from a degradation curve to a transition probability matrix can be performed. The formulae derived in this chapter are general and they can be applied to any degradation processes.

2.2 Repair Action Models

One of the advantages of the Markov Chain method is the flexibility in describing various action effects. Many time-dependent processes such as the protection of structures by coatings which themselves deteriorate or the interaction of two degradation mechanisms would be almost impossible to be described by analytic methods but are easily reproduced by the Markov Chain method.

The MR&R actions to be modelled for LCC analyses can be divided into the following groups:

- Mechanical concrete repair
- Patching
- Coating
- Realkalisation
- Electrochemical chloride extraction
- Cathodic protection

The modelling principles are shortly reviewed in the following sections.

2.2.1 Mechanical Concrete Repair

Methods of mechanical concrete repair mean methods in which all the concrete around the reinforcement is removed and replaced by new concrete. The placing of concrete may be done by conventional casting, shotcreting or other special concreting methods. A structural element that is replaced by a new prefabricated element can also be considered to be a method in this group.

A modelling of concrete repair includes:

- rules of states transition right after the repair and
- rate of degradation after the repair.

The rate of degradation after the repair can be evaluated by the same degradation models that were used for the original structure. However, as the quality of concrete, concrete cover etc. may be different from the original structure, the degradation rate of the repaired structure may have to be changed from the original.

Normally it is not possible to exactly tell the effects of a concrete repair on the condition state of a structure right after the repair but one can roughly evaluate the characteristic risks of repair methods related to poor concrete quality, poor bonding, surface cracking etc. As a result of these risks the expected state of the structure after the repair is not always the best possible but there is a probability that it ends up somewhere between the best state and the original state before the repair. So the expected state after the repair forms a probability distribution between the best state and the original state.

Sometimes a situation exists when there is not enough research data to build up the required action effect matrices. Then it may be reasonable to use a Delphi study for experts to evaluate the necessary transition probabilities. A team of experts will answer the question: "If a structure is before the repair at the state i , what is the probability that the structure will end up to the condition states $0, 1, \dots, i$ after the repair?" for each state i and for each repair method. The answers of the first round of a Delphi study are given back to the experts in the second round together with calculated means for evaluation and discussion. Usually the second session brings a fairly good consensus for the evaluated probabilities.

2.2.2 Patching

Patching is a repair method in which only a part of the surface area of a structure is repaired. The parts that will be repaired are the ones most severely suffered from degradation. The purpose of patching is not to repair the structure into its initial condition but only to extend the service life or the time until a heavier repair.

Patching is modelled in principle in the same way as the concrete repair but the repair is addressed only to the most severely degraded parts of the structure (e.g. the areas in condition state 3 or 4). In the programming of the degradation process it is possible to confine the fraction of the structural surface in which the degradation has exceeded a predefined limit state with a predefined probability and apply the transitions of condition state only to that part by a special patching matrix. The rest of the structure remains in its original condition state.

The rate of degradation after the repair in the patched areas is normally evaluated using the transition probabilities of the whole (original) structure as it is not possible to distinguish afterwards different parts of the structure.

2.2.3 Coating

Coatings are not considered to change the condition state of a structure. However, they change the rate of degradation.

Coatings have an effect on all the degradation factors of concrete such as carbonation, chloride contamination, frost attack and corrosion of reinforcement. The effect of coatings on chemical degradation is both direct and indirect. The direct effect is a result of the diffusion barrier produced by the coating between the structure and the environment. The indirect effect is a result of changes in the moisture content of the structure. Both effects must be considered to get a correct result. The effect of coatings on frost resistance and corrosion of reinforcement is mainly indirect, i.e. it depends on how the coating changes the moisture content of the subconcrete.

When evaluating the effects of coatings on the degradation of the structure, the degradation of the coating itself must also be considered. The degradation of the coating causes changes in the permeability of the coating thus changing both the direct and indirect effects of it on the structure.

Direct Effect of Coatings on Transition Probabilities

As proven in Chapter 2.1.4 the "drop-from-state" transition probabilities of degradation matrices are proportional to the rate of degradation at the corresponding interval of states. So by knowing the effect of coatings on the rate of degradation it is possible to evaluate their relative effect on the "drop-from-state" transition probabilities ($p_{i;i+1}$).

Consider that we can experimentally define a coating factor p_c that expresses the ratio between the rates of degradation of a structure as coated and as uncoated.

$$p_c = \frac{r_{coated}}{r_{uncoated}} \quad (24)$$

where

p_c is coating factor,
 r_{coated} rate of degradation as coated, and
 $r_{uncoated}$ rate of degradation as uncoated.

Then according to the rules presented in Chapter 2.1.4 we can claim that the transition probabilities for a coated structure can be determined from the following equation:

$$P_{i;i+1;coated} = p_c \cdot P_{i;i+1;uncoated} \quad (25)$$

To confirm this the following derivation is performed. The change in the fraction of structures in condition state i is as follows (ref. Chapter 2.1.4):

$$\Delta w_{i;coated} = w_{i;coated}^{t+1} - w_{i;coated}^t = w_{i-1}^t \cdot P_{i-1;i} \cdot p_c - w_i^t \cdot P_{i;i+1} \cdot p_c = p_c \cdot \Delta w_{i;uncoated} \quad (26)$$

where

$\Delta w_{i;coated}$ is change in the fraction of state i as coated, and
 $\Delta w_{i;uncoated}$ change in the fraction of state i as uncoated.

The average change in degradation per year can be expressed as:

$$\Delta DoD_{coated} = 0 \cdot p_c \Delta w_0^t + 1 \cdot p_c \Delta w_1^t + \dots + N \cdot p_c \Delta w_N^t = p_c \cdot \Delta DoD_{uncoated} \quad (27)$$

where

ΔDoD_{coated} is change in degradation of a structure as coated, 1/year, and
 $\Delta DoD_{uncoated}$ change in degradation of a structure as uncoated, 1/year.

ΔDoD expresses the change of degradation in a year and can thus be replaced by the derivative function of DoD or generally the rate of degradation r (ref. Chapter 2.1.4).

The coating factor p_c depends on the quality of the coating and also on the condition of the coating. As the condition of the coating changes with time also the value of p_c changes. We define the service life of a coating so that at the end of the service life its effect on the degradation rate of the structure is 0. So at the end of the service life of a coating the ratio of degradation rate between a coated and uncoated structure is 1. At the beginning of service life the corresponding ratio is expressed by the coating reduction factor, m_c . The value of m_c is between 0 and 1 and is dependent on the quality of a new coating. Between the start and the end of the service life this ratio is assumed to change from m_c to 1, proportionally to the probability function of the service life of the coating, $P_c(t)$. So at the moment of t from the start of a coating's service life the value of the coating factor can be evaluated as follows:

$$p_c = m_c + P_c(t) \cdot (1 - m_c) \quad (28)$$

where

m_c reduction factor for an undeteriorated coating,
 $P_c(t)$ cumulative probability distribution of the service life of a coating, and
 t age of a coating, year.

Combined with Equation 25 we get an expression for the transition probabilities of a coated structure:

$$P_{i;i+1,coated} = (m_c + P_c(t)(1 - m_c)) \cdot P_{i;i+1,uncoated} \quad (29)$$

According to Equation 29 all transition probabilities of a coated structure are time dependent as the transition probabilities change from year to year. The reducing effect of coatings on the "drop-from-state" probabilities is greatest right after the application or reapplication of a coating. By ageing of the coating these transition probabilities approach the values of uncoated concrete.

Equation 29 opens the way for combination of two degradation processes: 1) degradation of the coating, and 2) degradation of the structure. The combination is possible by parallel calculation of these degradation processes. In a life cycle table for the coating, the annual condition state distributions for the coating are determined. Next to it we have the LC table for the structure. Between these tables there are columns in which the new transition probabilities of the structure are determined according to Equation 29. The values of the cumulative probability function, P_c , are picked from the LC table of the coating corresponding to the limit state of the coating and are assigned to the equations of transition probabilities which are then applied to the determination of the condition state distributions of the structure.

The transition probability matrix of a coated structure would look as presented in Figure 18. First the transition probabilities for the "drop-from-state" probabilities are determined. Then the "remain-in-state" probabilities" are determined by subtracting the "drop-from-state" probabilities from 1.

State	0	1	2	3	.	N-1	N
0	$1 - p_c \cdot p_{01}$	$p_c \cdot p_{01}$	0	0	.	0	0
1	0	$1 - p_c \cdot p_{12}$	$p_c \cdot p_{12}$	0	.	0	0
2	0	0	$1 - p_c \cdot p_{23}$	$p_c \cdot p_{23}$.	0	0
3	0	0	0	$1 - p_c \cdot p_{34}$.	0	0
.
N-1	0	0	0	0	.	$1 - p_c \cdot P_{N-1;N-1}$	$p_c \cdot P_{N-1;N}$
N	0	0	0	0	.	0	1

Figure 18. Degradation transition probability matrix for a coated structure.

The above-presented method for combining the degradation of a coating to the degradation of a structure is general, i.e. it applies in principle to all degradation types. It can be applied to depassivation processes which are carbonation and the penetration of chlorides and the processes of active corrosion. It can also be applied to the degradation processes at cracks (Ref. Part II: "Reference Structure Models for Prediction of Degradation").

When applying this method to degradation processes it is important to observe, however, that the reduction factor m_c in these processes may be different. For instance, the reduction factor for carbonation is different from that for chloride penetration. For active corrosion it is usually assumed that a coating has no direct effect on the degradation rate ($m_c = 1$, as there is enough oxygen to continue corrosion in spite of the coating). However, a coating has a great indirect effect on the corrosion rate through the changed moisture content.

Indirect Effect of Coatings on the Transition Probabilities

The average annual moisture content of concrete under a new coating decreases to a level that is characteristic to the permeability probabilities of the coating. With increasing imperfection of the coating the average moisture content is increased until finally approaching the level of moisture content characteristic of an uncoated structure.

The average annual moisture content of a structure can be evaluated using the following equation:

$$w = w_{she} + m_{\varphi}(t) \cdot (w_{exp} - w_{she}) \quad (30)$$

where

w_{she} is average annual moisture content as sheltered from rain and splash, kg/m^3 ,
 w_{exp} average annual moisture content as exposed to rain and splash, kg/m^3 , and
 m_{φ} moisture factor of coating.

If the structure is not fully exposed to rain and splash (but is partially sheltered by other structures) the moisture content w_{exp} is replaced by w'_{exp} which is the real average moisture content of the structure without coating. Even in that case Equation 30 can be considered to be valid.

The moisture factor is evaluated from the equation:

$$m_{\varphi} = m_{\varphi 0} + P_p(t) \cdot (1 - m_{\varphi 0}) \quad (31)$$

where

$m_{\varphi 0}$ is moisture factor for a new coating ($t=0$), and
 $P_p(t)$ probability function of the coating service life.

The moisture factor for a new coating is evaluated from Equation 32. This equation was derived from the results of computer simulation [1]:

$$m_{\varphi 0} = \frac{m_{k0}}{0.6 \cdot m_{e0}^{0.6}} \quad (32)$$

where

m_{k0} is the relation of the capillary index of newly coated concrete to the capillary index of uncoated concrete,

m_{e0} the relation of the moisture transfer coefficient of newly coated concrete to the moisture transfer coefficient of uncoated concrete.

The coating properties of m_{k0} and m_{e0} are determined by simple water uptake and drying tests.

By changing the level of the moisture content in subconcrete the coatings have an impact on most degradation factors of concrete structures, such as carbonation, chloride penetration, corrosion of reinforcement and frost attack. The effect of coatings on the degradation rate can be evaluated approximately if the average annual moisture content can be predicted.

We assume now that the rate of degradation can be evaluated by the available degradation models 1) as sheltered from rain and splash, and as 2) exposed (or partially exposed) to rain and splash. The former rate of degradation corresponds to the moisture content w_{she} and the latter to the moisture content w'_{exp} . As a first approximation it is also assumed that the rate of degradation between these extremes changes in proportion to the moisture content.

So the rate of degradation of a coated structure can be evaluated from the following expression:

$$r_{coated} = r_{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (r'_{exp} - r_{she}) \quad (33)$$

where

r_{coated} is average rate of degradation of a coated structure,
 r'_{exp} average rate of degradation of a fully or partly exposed uncoated structure,
 r_{she} average rate of degradation of a fully sheltered uncoated structure, and
 $P_c(t)$ probability function of coating service life.

Based on the rule derived in Chapter 2.1.4 the "drop-from-state" transition probabilities are proportional to the rate of degradation. So it is assumed that the "drop-from-state" probabilities for a coated structure can be determined by the same kind of "interpolation" process as in Equation 33 for the rate of degradation:

$$p_{i;i+1}^{coated} = p_{i;i+1}^{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (p_{i;i+1}^{exp} - p_{i;i+1}^{she}) \quad (34)$$

The fact that this approach leads to the degradation rate presented by Equation 33, is further proven as follows:

$$\begin{aligned} \Delta w_{i,coated} &= w_{i,coated}^{t+1} - w_{i,coated}^t = w_{i-1}^t \cdot (p_{i-1;i}^{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (p_{i-1;i}^{exp} - p_{i-1;i}^{she})) \\ &- w_i^t \cdot (p_{i;i+1}^{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (p_{i;i+1}^{exp} - p_{i;i+1}^{she})) = \\ &= \Delta w_i^{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (\Delta w_i^{exp} - \Delta w_i^{she}) \end{aligned} \quad (35)$$

Applying this to the equation (which is equal to Equation 11):

$$\Delta DoD = 0 \cdot \Delta w_0^t + 1 \cdot \Delta w_1^t + 2 \cdot \Delta w_2^t + \dots + N \cdot \Delta w_N^t \quad (36)$$

it is easily shown that the change in degradation per year (= rate of degradation) is:

$$\Delta DoD_{coated} = \Delta DoD_{she} + (m_{\varphi 0} + P_c(t) \cdot (1 - m_{\varphi 0})) \cdot (\Delta DoD_{exp} - \Delta DoD_{she}) \tag{37}$$

So the indirect effect of coatings on the degradation of a structure can be taken into account by calculating the "drop-from-state" transition probabilities using Equation 34. To take into account also the direct effect of coatings the so modified transition probabilities are still multiplied by the coefficient p_c according to Equation 29.

The interpolation method as described above is general and applicable to any type of degradation. If the degradation models for carbonation and chloride contamination include the accelerating impact of frost attack there is no need to study the impact of coatings on frost attack separately.

The calculation method of coating effects is demonstrated in Figure 19. The LC table of a coating is on the left and the LC table of the structure is on the right side of the figure. The necessary calculations for modified transition probabilities are presented between these tables. A graphic representation is given in Figure 20.

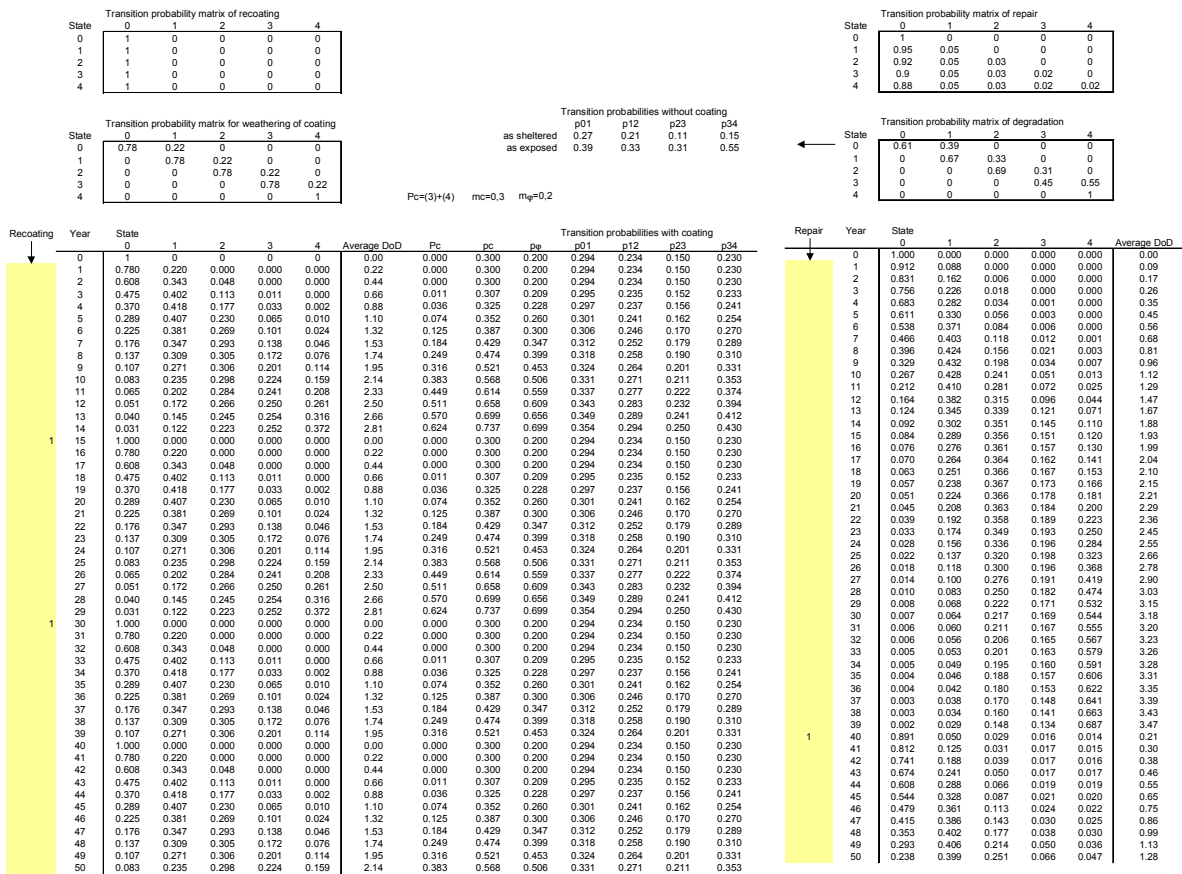


Figure 19. The determination of coating effects on the degradation of a structure.

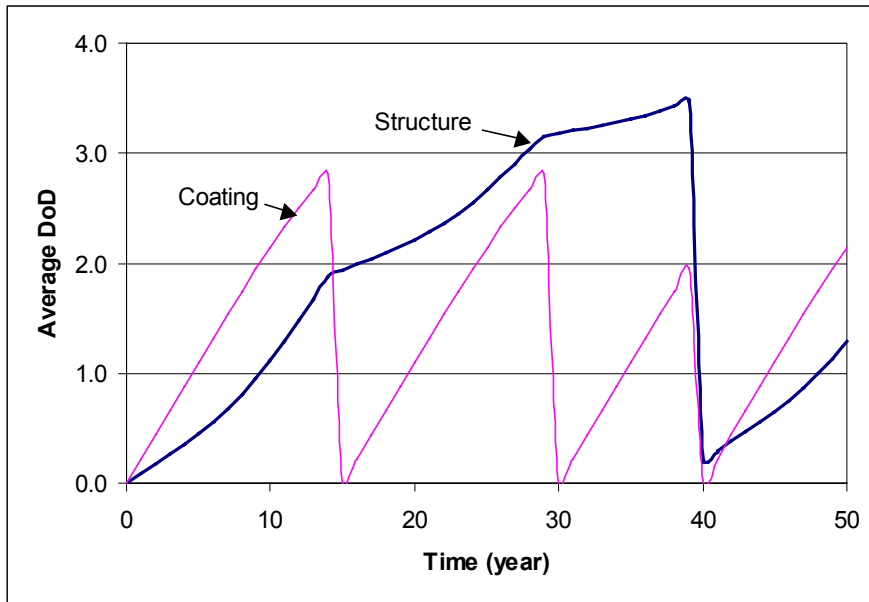


Figure 20. Average DoD of the coating and the structure.

2.2.4 Concrete Overlayer

Direct effect

If a layer of concrete is placed on the surface of a concrete structure, it can be considered to behave as a coating. So the layer does not immediately change the condition state of the structure but it protects it from further degradation. Analogically with coatings, it is possible to determine the rate of degradation from the following equation:

$$r_{protected} = (m_p + P_p(t) \cdot (1 - m_p)) \cdot r_{unprotected} \quad (38)$$

where

$r_{protected}$ is the rate of degradation in a structure which is protected by a concrete overlayer,
 $r_{unprotected}$ the rate of degradation in the same structure without concrete overlayer,
 m_p reduction factor of degradation for a new concrete overlayer, and
 P_p probability function of the concrete overlayer.

The service life of an unreinforced layer of concrete can be evaluated based on frost attack only. This can be done by a separate LC table using frost attack models (ref. LIFECON Deliverable 2.1 D). The probability function P_p in Equation 38 refers to the service life function determined from this table.

During the service of the concrete overlayer the protective properties of the layer changes. At the start of the service life when no carbonation or chloride penetration has occurred the layer can be considered to completely protect the substructure from further carbonation or chloride penetration. So $m_p \approx 0$ at the start of the service life.

After complete carbonation of the overlayer the following theoretical formula can be derived for m_p [2]:

$$m_{p;ca} = \frac{\sqrt{1 + 2 \frac{R_b}{x_1}}}{1 + \frac{R_b}{x_1}} \quad (39)$$

where

$m_{p;ca}$ is the reduction factor of carbonation for a protective concrete layer depending on the relation R_b/x_1 ,

x_1 depth of carbonation behind the concrete layer, m,

R_b is equivalent thickness of the concrete layer ($= \frac{\delta_b}{\delta_p} \cdot h$),

where

δ_b is coefficient of diffusion of concrete in the structure with respect to carbon dioxide, m^2/s ,

δ_p coefficient of diffusion of concrete in the overlayer with respect to carbon dioxide, m^2/s , and

h thickness of the layer, m.

According to Equation 39 the value of the reduction factor depends on the depth of carbonation in the structure. It is assumed that an average value for the reduction factor can be obtained by assigning for the carbonation depth $x_1 \approx 10$ mm and that the coefficient of diffusion is inversely proportional to the nominal compressive strength of concrete. Accordingly we get for the reduction factor is determined as:

$$m_{p;ca} = \frac{\sqrt{1 + 2 \frac{K_p}{K_b} \cdot \frac{h}{10}}}{1 + \frac{K_p}{K_b} \cdot \frac{h}{10}} = \frac{\sqrt{1 + 2 \cdot \frac{K_p}{K_b} \cdot \frac{h}{10}}}{1 + \frac{K_p}{K_b} \cdot \frac{h}{10}} \quad (40)$$

where

K_p is nominal strength of concrete overlayer, MN/m^2 ,

K_b nominal strength of structural concrete, MN/m^2 , and

h thickness of overlayer, mm.

As an example, if the nominal strength of concrete is K35, the nominal strength of shotcrete is K45 and the thickness of the overlayer is 20 mm, then the reduction factor is calculated as:

$$m_{p,ca} = \frac{\sqrt{1 + 2 \cdot \frac{45 \cdot 20}{35 \cdot 10}}}{1 + \frac{45 \cdot 20}{35 \cdot 10}} = \frac{2.48}{3.57} = 0.69 \quad (41)$$

This is the reduction factor of the overlay after complete carbonation. Assumably this formula can also be used for those cases in which the chloride penetration is the determinative factor for the depassivation time. It means that the reduction factor for the penetration of chlorides after complete contamination of the overlay by chlorides (when the depth of the critical chloride content is greater than the thickness of the layer) is constant and can be evaluated by the same equation.

The structure that is protected by a layer of concrete can be additionally protected by a coating. In a system of a coating and a concrete overlay the coating has an influence on the condition of 1) the overlay and 2) the structure. The effect on the concrete overlay is evaluated by similar methods as those presented for the coating effects on the structure (see above). The coating retards the processes of carbonation, chloride penetration and frost attack in the concrete overlay. Thus it decreases the values of the probability function of the concrete overlay, P_p .

The effect of a system coating + overlay on the degradation of a structure can be evaluated from the following equation:

$$p_{c+p} = (m_c + P_c(t) \cdot (1 - m_c)) \cdot (m_p + P_p(t) \cdot (1 - m_p)) \quad (42)$$

where

- p_{c+p} is system factor for coating + overlay,
- $P_c(t)$ service life probability function of the coating,
- $P_p(t)$ service life probability function of the concrete overlay,
- m_c reduction factor for coating, and
- m_p reduction factor for concrete overlay.

The transition probability matrix presented in Figure 18 applies with the exception that p_c is replaced by p_{c+p} .

The above-presented general rules can be applied to any degradation types. However, the reduction factors for the concrete overlay change depending on the degradation type. Both theoretical and experimental studies can be used to determine the correct values for the reduction factors.

Indirect Effect

The concrete overlay also has an indirect effect on the degradation rate of a concrete structure as the moisture stress in the structure is reduced. The thicker is the layer, the smaller is the moisture stress in the subconcrete.

In principle the effects of a concrete overlayer on the moisture stress and degradation of the structure are evaluated using the same methods as those presented for coatings. The only difference is that the factor $m_{\varphi 0}$ for a concrete layer can not be evaluated by Equation 32. It is known, however, that the moisture content in a structure is rapidly reduced with the distance from the surface [3]. So an experimental function is tentatively given for the moisture reduction factor of a concrete overlayer in Equation 43. The value of the factor is graphically presented in Figure 21.

$$m_{\varphi 0;overlayer} = \frac{1}{\exp(0.00005 \cdot h^3)} \quad (43)$$

where

h is thickness of the concrete overlayer, mm.

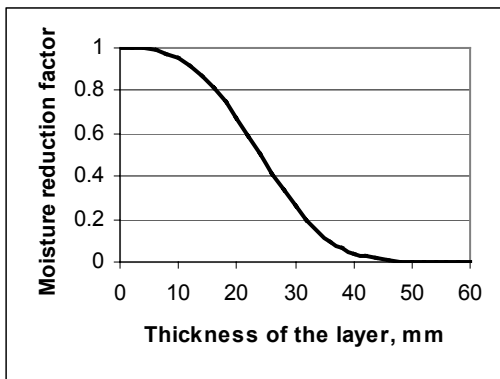


Figure 21. Moisture reduction factor of a concrete overlayer as a function of layer thickness (tentative).

In a combined system of a coating and concrete overlayer the "drop-from-state" transition probabilities are determined from the following equation:

$$p_{i;i+1} = p_{i;i+1}^{she} + (m_{\varphi 0;coating} + P_c(t) \cdot (1 - m_{\varphi 0;coating})) \cdot (m_{\varphi 0;overlayer} + P_m(t) \cdot (1 - m_{\varphi 0;overlayer})) \cdot (p_{i;i+1}^{exp} - p_{i;i+1}^{she}) \quad (44)$$

So the principle of "interpolation" is used in the same way as for coatings alone. To also take the direct effects into account the so determined transition probabilities are still multiplied by the "system factor", p_{c+p} , in Equation 42.

2.2.5 Cathodic Protection

The idea of the cathodic protection (CP) method is to provide the structure with an extra electrode to feed a protective current for the reinforcement. As a result of the protective current the rate of corrosion is considerably reduced if not completely stopped in the reinforcement.

It is essential that the protective current is evenly distributed over the surface of the structure. This is possible by an electrically conductive layer or a coating spread over the structure or a

noble metal mesh that is placed in a layer of concrete on the structure. The latter method is usually applied.

The protective current in the CP method has an effect on the degradation rate of the structure only in the active corrosion phase. So, only the transition probabilities for the active corrosion are changed as a result of this method. As the rate of degradation at the active corrosion phase is directly dependent on the corrosion current, the "drop-from-state" transition probabilities are assumed to be reduced proportionally to the reduction in corrosion current. So the transition probabilities for the states of active corrosion are determined as follows:

$$P_{i;i+1,protected} = a \cdot P_{i;i+1,unprotected} \quad (45)$$

where

a is reduction factor of corrosion current showing the relation between the corrosion currents as protected and unprotected.

Normally the corrosion current cannot be completely stopped by the CP method. So the reduction factor a is assumed to be greater than 0.

When the anode mesh has been placed into a concrete layer on the surface of a structure it is obvious that the concrete layer also has a protective effect. The protective impact of the concrete layer can be evaluated using the method presented in the previous chapter for a protective overlayer.

2.2.6 Realkalisation

Since corrosion in carbonated concrete is a result of disruption of the passivating film at low pH levels, a natural way to solve the problem of corrosion is to raise the pH of concrete sufficiently to reform the passivating film. In the electrochemical realkalisation method, alkaline ions are introduced into concrete by electric current for this purpose. A temporary electrode is installed on the surface of the concrete structure embedded in an electrolytic mass containing a solution of sodium or potassium carbonate.

The alkalinity of the sodium or potassium carbonate in the structure is high enough to repassivate the reinforcing steels and protect them from corrosion. As the solutions are already carbonated they cannot be recarbonated. So the protection is supposed to be long lasting.

Although there are no exact methods to evaluate the rate of reneutralisation of the protected surfaces it would be most appropriate to evaluate it as a fraction of the original rate of carbonation. Accordingly the "drop-from-state" transition probabilities for carbonation are reduced as presented in Equation 46.

$$P_{i;i+1,protected} = b \cdot P_{i;i+1,unprotected} \quad (46)$$

where

b is reduction factor of neutralisation rate showing the relation between the rate of neutralisation after protection by the realkalisation method and the rate of carbonation as unprotected.

The realkalisation method is assumed to have no effects on either the condition state of the structure with respect to chloride contamination or the rate of chloride penetration. The symptoms of frost attack cannot be repaired by this method either.

2.2.7 Electrochemical removal of chlorides

Corrosion in chloride-contaminated concrete leads to disruption of the passivating film. If the chlorides around the steel could be removed and a sufficiently high pH could be provided in concrete to re-establish the protective film, the problem of corrosion would be solved. Traditionally this has been done by mechanical repair methods in which all the concrete around the steel is removed and replaced. A more environmentally-friendly alternative is electrochemical desalination.

As in the realkalisation, in the electrochemical desalination electric current is also used to change the chemical environment around the reinforcement. However, as the treatment duration in the realkalisation method are typically between 3 to 7 days, the treatment duration in the electrochemical desalination vary from 3 to 7 weeks. Even with such long treatment times the structure cannot be completely cleaned from chlorides. A fraction of chlorides is always left in the structure. This fact should be considered in the action effect matrices for realkalisation.

After the repair the rate of chloride penetration can be assumed to be unaffected as compared to the original rate of penetration in the structure.

The method of electrochemical removal of chlorides is assumed to have no condition-related effects with respect to carbonation or frost attack.

2.3 Matrix Models for Sequential Degradation Processes

In cases when degradation mechanisms consist of a sequence of processes it is advisable to combine these mechanisms and use a combined rating system for them. This allows treating both mechanisms in the same LCC analysis. The Markov Chain mathematics supports this kind of combined systems.

For concrete structures a combined rating system can be defined for the total process of corrosion of reinforcement. This consists of (1) depassivation by carbonation or penetration of aggressive chlorides, and (2) active corrosion causing volume increase of rebar, cracking, spalling etc.. The combination is performed so that the limit state of depassivation is defined as being the same as the starting state of the active corrosion.

The combination of depassivation and active corrosion into the same LCC analysis makes it possible to design the complete life cycle action profile (LCAP) by the same LCC analysis. So it

is possible to treat the preventive actions for carbonation and chloride penetration, such as coatings and impregnations, in the same LCC analysis together with structural repair actions.

If frost attack occurs together with corrosion of reinforcement so that it is difficult for an inspector to identify the primary cause of damage it may be advantageous to combine also frost attack to the same system of "surface damage". Then frost attack is assumed to affect both the processes of carbonation and penetration of chlorides and the cracking and spalling of the concrete cover due to corrosion of reinforcement. Although frost attack increases the rate of these processes the state definitions of carbonation, chloride penetration or corrosion of reinforcement do not have to be changed. (The combination of frost attack into the models of depassivation and corrosion of reinforcement has been worked out in Part II: "Reference Structure Models for Prediction of Degradation").

Quite independent of the frost attack, the following measures have to be taken for combination of depassivation and active corrosion into the same scale:

1. Combination of the initial distributions (obtained from inspection and condition assessment of depassivation and active corrosion) into a combined distribution and
2. Combination of degradation matrices for depassivation and corrosion of reinforcement into a degradation matrix for the combined system.

2.3.1 Combination of Initial Distributions

In the combination of the initial distributions for depassivation (carbonation or chloride penetration) and corrosion of reinforcement it is advisable to proceed with the following stages:

- Choose the more determinative mechanism between carbonation and chloride penetration (the most determinative mechanism is referred as the depassivation mechanism).
- Combine the depassivation mechanism and the mechanism of corrosion into a 8-state system using the instructions in Figure 22.
- (Optionally) reduce the 8-state system into a 5-state system according to the instructions in Figure 23.

At the stage 1 the more determinative mechanism is often self evident from the environmental burdens that the structure is exposed to.

At stage 2 the combination into a 8-state system is performed as follows: Consider the condition state distributions for depassivation and corrosion represent the same structure or a group of structures. As a structure cannot be corroded unless it is depassivated first (in very unusual cases corrosion without depassivation is possible, but as a rule it is impossible) there cannot be overlapping between depassivation states 0, 1, 2 and corrosion states 1, 2, 3 and 4. So these states can be directly assigned to the combined system as presented in Figure 22. The total of states 3 and 4 for depassivation must be greater than the total of states 1, 2, 3 and 4 for corrosion. So the state 3, "state of depassivation without corrosion", in the combined system can be determined by subtracting the total of corrosion states 1, 2, 3 and 4 from the total of depassivation states 3 and 4. Then the total of fractions of all states in the combined system equals to 1.

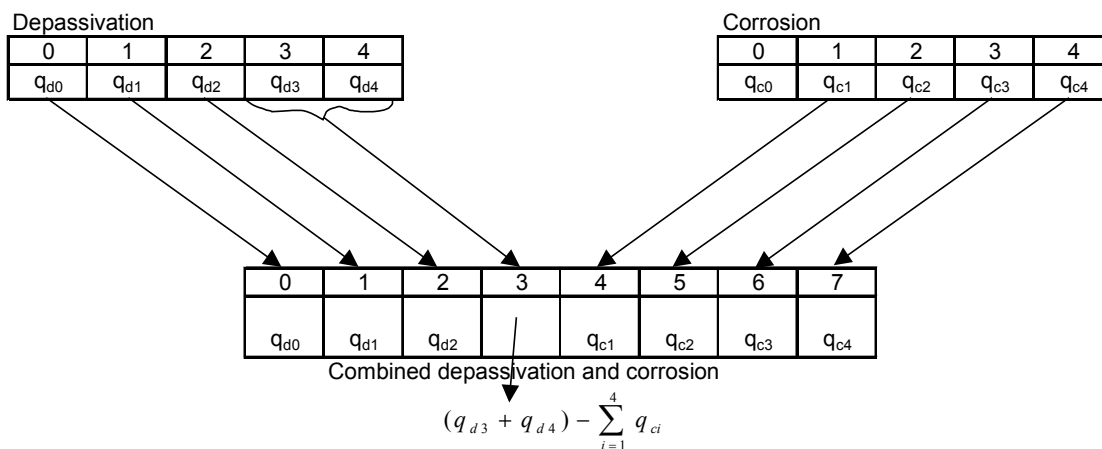


Figure 22. Method for combining depassivation and corrosion into a common 8-states rating system.

In Figure 23 we see a method for reducing the 8-state system into a 5-state system. In this case state 5 in the 8-state system was chosen as the limit state in the 5-state system (= state 3). The fraction of state 1 is divided and a half of it is added to both the states 0 and 2 to make the states 0 and 1 in the new 5-state system. Likewise the fraction of state 4 is divided and a half of it is added to both the states 3 and 5 to make the states 2 and 3 in the 5-state system. States 6 and 7 are merged and assigned to the terminal state 4 in the new system.

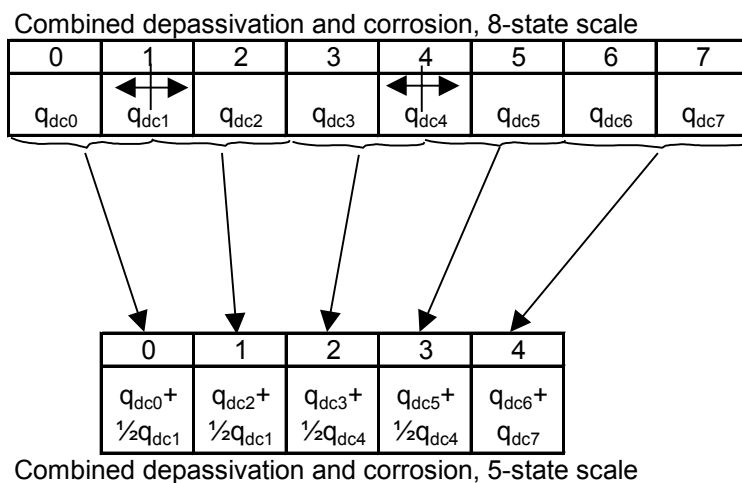


Figure 23. Combining depassivation and corrosion into a 5-states rating system.

2.3.2 Combination of Matrix Models of Degradation

Consider that originally we have matrix models created for both depassivation and corrosion of reinforcement (both in a 5-state system 0, 1, 2, 3, 4). Now we want to produce a matrix model for a combined 8-state system. The degradation matrix for the combined system can be obtained only by compiling the elementary degradation matrices as presented in Figure 24. The transition

probabilities of the depassivation matrix corresponding to state 4 are neglected (in general for states greater than N_0 are not needed) and the transition probability p_{33} (in general p_{N_0, N_0}) is replaced by the transition probability p_{00} of the matrix for active corrosion matrix. The other transition probabilities of the active corrosion matrix are placed in their respective positions as unchanged. Only the index numbers of these transition probabilities will be changed according to the new state numbering.

State	0	1	2	3	4	5	6	7
0	p_{00}	p_{01}	0	0	0	0	0	0
1	0	p_{11}	p_{12}	0	0	0	0	0
2	0	0	p_{22}	p_{23}	0	0	0	0
3	0	0	0	p_{00}	p_{01}	0	0	0
4	0	0	0	0	p_{11}	p_{12}	0	0
5	0	0	0	0	0	p_{22}	p_{23}	0
6	0	0	0	0	0	0	p_{33}	p_{34}
7	0	0	0	0	0	0	0	1

Figure 24. Combined transition probability matrix for depassivation and corrosion in a 8-state system. The dotted lines confine the matrix for depassivation and the double lines the matrix of active corrosion.

To reduce the matrix for a 5-state system according to the system presented in Figure 23, the original matrix models have to be modified. If the definition of the limit state is not changed but only the state number is changed (as in the case of depassivation in our example where N_0 is changed from 3 to 2) then only the scaled degradation function is modified by Equation 7. However, if the definition of limit state is redefined (as in the case of corrosion of reinforcement in our example) the degradation function f must be first redefined according to the new limit state definition. Assuming that in the original distribution of corrosion the amount of corrosion at state 2 is $2/3$ from the limit state definition ($=3$) of maximum allowable amount of corrosion, the new degradation function f_1' in relation to the original degradation function f_1 would be related as 3:2 (ref. Degradation models 4.5.2.).

Starting from the original degradation functions f_0 and f_1 the new scaled degradation functions would be:

Depassivation: $\text{DoD} = 2 f_0$ (ref. Equation 7, $N_0 = 2$)

Active corrosion: $\text{DoD} = 3/2 f_1$ (ref Equation 9, $N_0 = 1$)

The elementary degradation matrices for the new degradation functions are then produced. For depassivation in this case a 4x4 matrix is obtained and for active corrosion a 3x3 matrix.

The final degradation matrix for a combined 5-state system is now produced by "compiling" in the same way as for the 8-state system. The matrix for active corrosion is placed above the matrix of depassivation so that the transition probability p_{22} for depassivation is replaced by p_{00} for active corrosion and the rest of transition probabilities for corrosion are arranged accordingly. The rows and lines for state 3 in the original matrix for depassivation are ignored. Figure 25

shows the way of compiling the matrices. As the state 4 is the greatest state in the combined system the transition probability p_{44} is assigned to 1.

State	0	1	2	3	4
0	p_{00}	p_{01}	0	0	0
1	0	p_{11}	p_{12}	0	0
2	0	0	p_{00}	p_{01}	0
3	0	0	0	p_{11}	p_{12}
4	0	0	0	0	1

Figure 25. Combined transition probability matrix for degradation. The dotted lines confine the matrix for depassivation and the double lines the matrix of active corrosion.

3 Markov Chain Based Life Cycle Cost Analysis

3.1 General

Is it feasible to use a coating or some other protective maintenance method to retard the degradation? Is it more cost-effective to use an electrical repair method (realkalisation or electrical removal of chlorides) than a mechanical repair method? Among different coating systems which one should be preferred? How can the eventual upkeep of structures be optimised to take into account all the different requirements set in the maintenance policy?

In LIFECON LMS all these and many other problems are studied using the Markov Chain based LCC analysis which is a tool for financial optimisation of structures. The idea of the Markov Chain LCC analysis is to combine the Markov Chain life cycle table with the traditional LCC cost analysis table the principles of which are presented in international standards ISO 15686-5 [4] and ASTM E 917 [5]. An example of a traditional life cycle cost analysis is presented in Deliverable 5.2.

In the Markov Chain LCC analysis the condition of a structure is reproduced taking into account both degradation and the effects of MR&R actions during a desired time frame so that the condition requirements are fulfilled and to calculate the MR&R costs accordingly during that time frame. Other costs such as user costs, risk costs, environmental costs (impacts) can be integrated to the LCC analysis to take into account other aspects in the decision making on the maintenance strategy. The final purpose of the LCC analysis is to find the most feasible and economically most effective maintenance strategy to upkeep the structures. It is an optimisation problem where the user seeks to find the most effective life cycle action profile (LCAP) for each structural part and for the defined period of time.

In the Markov Chain based LCC analysis the whole life cycle action profile during the given time frame is reproduced as a series of annual condition state distributions. There are two ways for the definition and timing of MR&R actions: automatic and manual. In the automatic LCC analysis the definition of MR&R actions is performed automatically using the decision tree method and the timing of MR&R actions is defined automatically using the pre-set condition criteria. So the automatic LCC analysis is able to guard that the pre-set condition criteria are never exceeded. In a manual analysis the definition and timing of MR&R actions are done manually. In principle a designer can specify whatever actions are to be applied at any time during the time frame. However, even in the manual analysis method many automatic features are available.

3.2 Principles of LCC Analysis

3.2.1 LCC Analysis Table

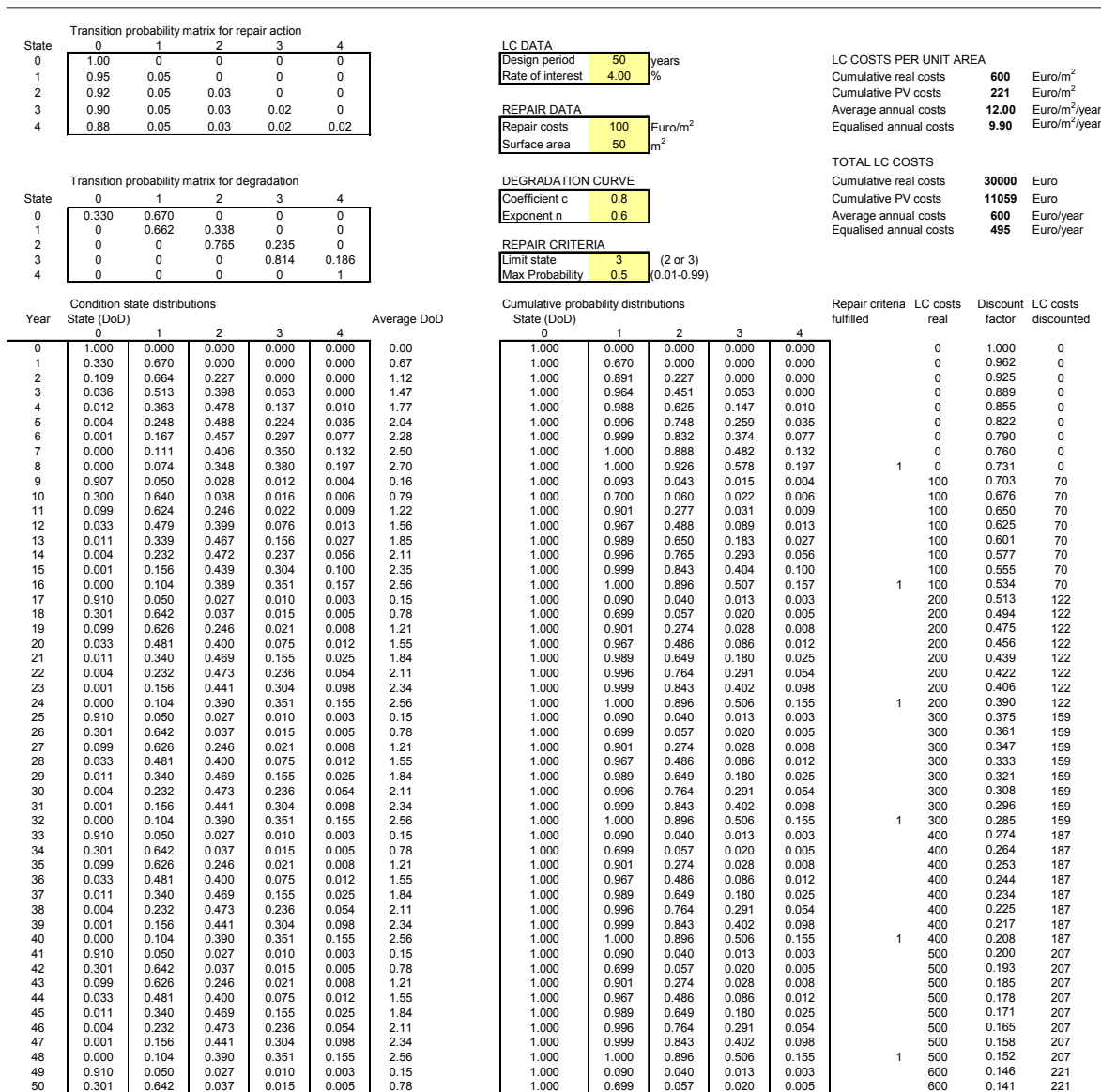
Figure 26 shows an example of a simple LCC analysis table. In this analysis it is assumed that there are no coatings or concrete protection used on the structure and only one repair method is defined. In a real LCC analysis table many optional MR&R methods can be used for the upkeep

of the structure. However, this simple example serves as a model for illustrating the calculation principle.

On the left side of the figure the Markov Chain LC Table is shown for the structure. The table is programmed so that when the parameters of the degradation curve are changed (yellow cells for the coefficient c and the exponent n) the transition probabilities for degradation are automatically changed and the whole LC Table is changed. Also the columns for average DoD and cumulative distributions are changed respectively.

All the yellow cells can be changed by the designer. The top most cells are for defining the design period (=time frame for the LCC analysis) and the rate of interest (used for the determination of present value costs). The design period is only limited by the length of the table (see the years along the left side of the MC Table). The length of a real analysis table may be 500 years or even longer.

The rate of interest is a parameter that has great influence on present value (PV) costs. In business economics the rate of interest is defined according to the lending interest. For public investments and social economical calculations the definition of interest rate is more difficult. Actually there is no universally accepted method for it but there may be national recommendations. The range of opinions and recommendations usually vary from 1 - 6 %.



In the cost counters at the right side of Figure 26 the repair costs are summed to cover the costs of the whole design period. The costs for one repair depend on the unit costs and the area of repair. In this simple analysis table these quantities are input data which a designer can specify. The total costs are calculated as real costs and as present value costs. The summary of costs is presented in the upper right corner of Figure 26.

Many kinds of maintenance and repair actions can be included in a life cycle of a structure. So Figure 26 is inadequate to represent a real life cycle analysis. For instance, the degradation can be affected by placing an extra layer of concrete (or mortar) or by applying a coating on the structure. However, both the extra layer of concrete and the coating deteriorate themselves. So before evaluation of their effect on the condition of the structure, the condition of the concrete layer and the coating must be first evaluated. In practice three calculation tables of the form presented in Figure 27 are needed: 1) table of coatings 2) table of extra concrete layer and 3) table of the structure. These tables are connected to each other by rules which take into account the mutual condition related effects of possible coatings and protective concrete layers and the original structure, as schematically presented in Figure 30.

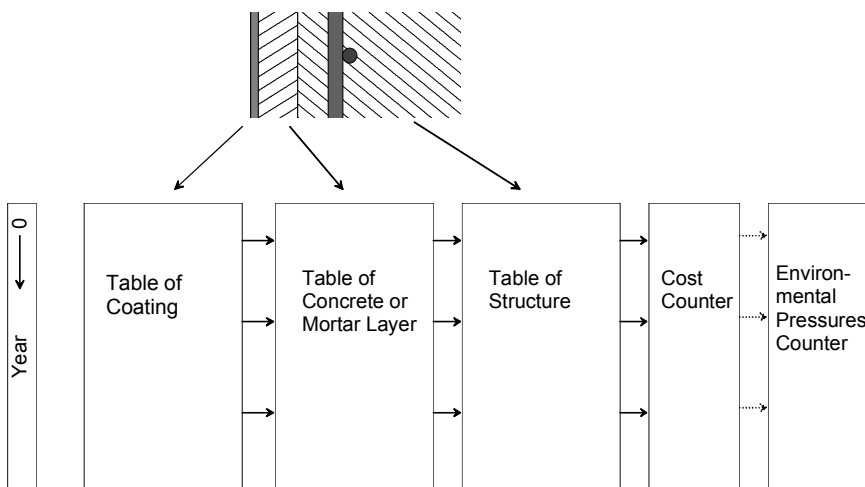


Figure 27. Markov Chain LC Tables of coatings, concrete or mortar layers and the structure connected to each other and counters for costs and environmental pressures.

3.2.3 Whole layout of Markov Chain Based LCC Analysis

To build up an automatic LCC analysis many kinds of initial data are required. The data are stored in different tables around the core tables which are the Markov Chain LC Table and the cost counters (ref. chapter 3.2.2). Figure 28 shows the general layout of an automatic Markov Chain LCC analysis table. The parts of the analysis table are:

1. Tables for initial data of objects and components
2. Tables for MR&R systems
3. Tables for definition of actions
4. Guiding columns for the Markov Chain LC Tables
5. Markov Chain LC Tables
6. Cost counters

7. Results Tables and Diagrams

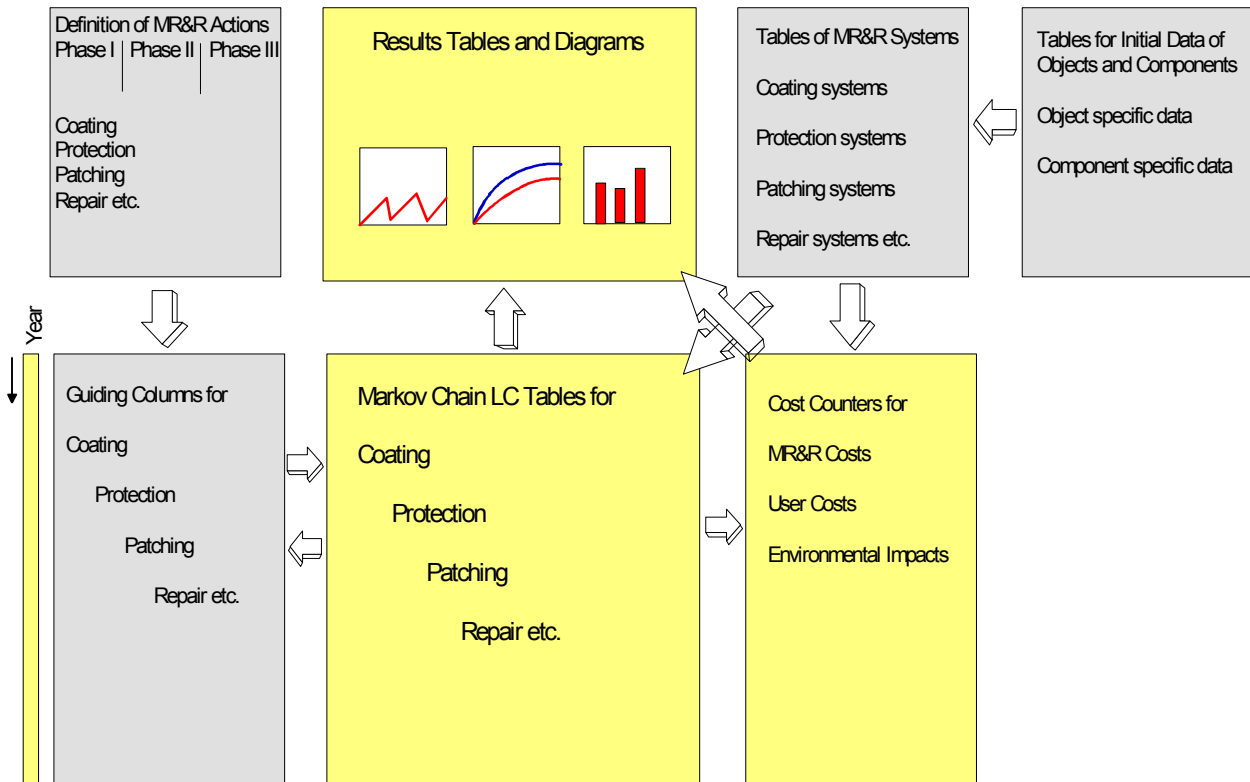


Figure 28. Layout of a Markov Chain LCC analysis table.

3.2 Tables of Initial Data for Objects and Components and Table of MR&R Systems

When starting a LCC analysis programme the designer first defines the object to be analysed according to the defined modular breakdown of structures. When the designer selects the object (which is the structure itself, e.g. a bridge) all register data pertaining to that object are available for the analysis. Next the module, component, subcomponent etc. are selected. All the data of these structural parts are then available to be used as parameters for models of degradation, effect action and costs and for other needs of the analysis. These data also include all the necessary information for evaluating the environmental stresses at both the macro level and the micro level. If not data for the specific object or component are available, default data provided by the programme are used instead.

When using a spread sheet analysis programme (as in the examples of this report) it may be difficult to operate directly with the database. So, it may be convenient first to build up the object and component specific initial data tables on a spread sheet by a programme which can read the database data and store it into a spread sheet table. The collection of initial data into spread sheet tables can be performed as a batch processing quite independently from the LCC analysis programme.

The Tables for initial data of objects and components consist of object and component specific data that are required in the LCC analysis. The object level data comprise of the following data types:

1. Identification data
2. Dimension data
3. Environmental burden data
4. User cost data
5. etc.

The component specific data consist of the following data types:

1. Identification data
2. Dimension data
3. Structural data
4. Data on previous MR&R actions
5. Inspection and condition assessment data
6. etc.

The Tables of objects and components contain also the default values which are used in the analysis if specific data is not available.

The Table of MR&R systems contains all data pertaining to MR&R systems used in the management of structures. The system data is categorised as follows:

1. Coating systems,
2. Concrete overlay systems,
3. Patching systems
4. Repair systems, and
5. Renovation systems.

Each MR&R action category contains several MR&R systems (or methods). Accordingly the category of coatings is comprised of several coating systems. Concrete protection category refers to methods in which a layer of shotcrete, conventional concrete or cement mortar is applied on the whole surface of the structure. Cathodic protection methods with a net anode embedded in a layer of concrete on the original structure is also included in this category of actions.

The category of structural repairs refers to MR&R actions by which the concrete around the reinforcement is renewed. This can be done by removing and replacing concrete around the steel bars by mechanical repair methods. Electrochemical methods such as realkalisation and chloride extraction are included in this category as the concrete environment around the reinforcement is renewed.

Patching means partial repair of the most attacked areas of the structure, coating or concrete protection. The methods of structural patching are comparable to structural repair in that they also change the environment around the reinforcement. However, this is only done locally and the other parts of the structure being remained unchanged. So patching is not considered as starting a new service life but only as extending the on-going service life.

Renovation refers to complete replacement of a component by a new one. The component can be reconstructed at site or a new prefabricated element can be installed in place of the old component.

Each MR&R system has a code number in the left column of the table. The data of a specific MR&R system is identified by that code and the data pertaining to the system is situated in the row indicated by the code number. The structure of the Table of MR&R systems is schematically presented in Figure 29.

	Code	Material and consumption data	Cost data	Ecological data	Degradation data	Action effect data
Coating systems	1					
	2					
	3					
	4					
Concrete overlayer systems	1					
	2					
	3					
	4					
Repair systems	1					
	2					
	3					
	4					
Renovation systems	1					
	2					
	3					
	4					

Figure 29. Schematic presentation of the Table of MR&R systems.

The Table of MR&R systems is more than just a storage of data. All models, i.e. cost models, degradation models and action effect models, are programmed in the Table of MR&R systems as the unit costs, rate of degradation and the action effects are specific to the MR&R systems. So, the system table consist of model formulae and their parameters.

The data of the Table of MR&R systems can be classified as follows:

1. Material and consumption data
2. Cost data
3. Ecological data
4. Degradation data
5. Action effect data

The material and consumption data consists of characteristic data of the system. Most of them such as material composition, layer thickness, strength, permeability etc. are parameters of the cost and degradation models. These data are often linked from the Tables for initial data of objects and components and on special data tables for coatings and concretes.

The cost data in the Table of MR&R systems result in the evaluation of unit cost of a MR&R system. The unit cost may depend on many parameters such as the depth of repair and the area of repair. If the unit cost depends on the condition of the structure, the reduction coefficient for

condition cannot be determined in the table of MR&R systems but is added in the final cost calculation when the condition of the structure at the moment of repair is known.

Ecological data is determined in principle in the same way as the costs. The environmental impact data per functional unit is determined for the MR&R system applied. These include data on the consumption of energy, emissions into air and non-renewable raw materials. The determination of environmental impact data is performed based on the consumption of materials and the environmental profiles of materials which are given in auxiliary tables of materials.

The degradation data include environmental burden data, degradation models and conversion models to transition probabilities. The environmental burden data and some material and structural parameters for the degradation models are linked from the Table for initial data of objects and components. The inspection and condition assessment data is also of importance as the degradation models can be calibrated using this data.

The action effect data consists of the evaluated transition probabilities to be applied every time when the MR&R system is applied.

3.3 Definition of actions

The definition of actions can be performed manually or automatically. In an automatic process the MR&R actions are defined by the decision tree method. In a manual analysis the designer makes the choices for MR&R actions manually.

The definition of actions within each MR&R action category is done by answering the questions posed in Table 2. The questions and answers are further described in the following paragraphs.

Table 2. Definitions of actions.

	Question	Answer
1	Is the MR&R action group used during the design period?	Yes/no
2	System?	Code of the MR&R system within the MR&R action group
3	Limit condition state?	Limit state for the action, e.g. 3 or 4
4	Maximum allowable probability for exceeding the limit state?	Probability as % (exceeding the given percentage will trigger the action)
5	Maximum number of repeated actions?	Number of allowable repetitions of an action before a heavier action.

Answering "yes" means that the specified MR&R action category is potentially used during the design period. It does not guarantee an automatic application of this MR&R action category because the conditions presented later in the question list must also be fulfilled. However, answering "no" precludes unconditionally the application of that action group.

The code number of the system refers to the specific MR&R system presented in the Tables of MR&R systems. For example in the case of the coating group it refers to a specific coating system (with defined materials and material thicknesses). In the case of concrete protection group, it refers to specific concrete or cathodic protection systems (with defined materials, thicknesses and techniques). In the case of concrete repair the system code refers to a specific repair system (with defined materials and casting techniques and electrochemical repair systems).

The maximum allowable probability sets the maximum limit for exceeding the limit state. It can be interpreted as expressing the maximum allowable fraction of the surface area of a component to exceed the limit state. The MR&R actions for structures are automatically triggered when the maximum allowable probability for the defined limit state is exceeded.

When specifying the maximum allowable probability all possible risks that the exceeding of the limit state would entail should be considered: safety of the structure, security of people, economic and environmental risks, serviceability, aesthetic and cultural values, etc.. The site of the structure, whether in urban or rural district, may also have an influence on the maximum allowable probability.

In the case of patching the definition of the limit state has a special meaning. In patching methods the repair actions are addressed only to the areas where the limit state is exceeded at the time of repair. The rest of the structure (at better condition states) will remain untouched.

The Markov Chain LC table "remembers" the previous actions and counts the sequential actions of the same type. So a designer can define the maximum number of actions before a heavier action is applied. For instance, it is possible to define the maximum number of patchings before a structural repair etc.

The MR&R action categories are independent on each other. It means that several systems which are not from the same action category can be applied at the same time. So it is possible to apply e.g. a coating together with structural repair or coating and concrete protection together with structural repair. However structural patching and structural repair cannot be scheduled for the same year.

The lifetime of a concrete component is considered to be composed of the three phases for which the MR&R actions may be defined independently:

- Phase I Residual service life of the component.
- Phase II From the end of the residual service life to the end of the residual life cycle of the component.
- Phase III From the end of the on-going life cycle to the end of the last life cycle.

As a concrete component can be repaired completely without replacing the whole component (by completely replacing the deteriorated concrete surfaces), a new service life of the component is considered to start from the repair. Possibly even many consecutive repairs can be accepted before the component must be replaced. Thus the life cycle of a component is not considered to

end until it is completely renovated (replaced by a new one). Accordingly, a structural repair generates a new service life and a renovation generates a new life cycle for the component.

The length of a service life of a component can be extended by patchings and protections. By protections it is meant coatings, concrete or mortar protections, cathodic protection etc.. Each protection has a service life of its own which is shorter than or equal to the service life of the component. So the life cycle can be described as a combination of nested arches as presented in Figure 30.

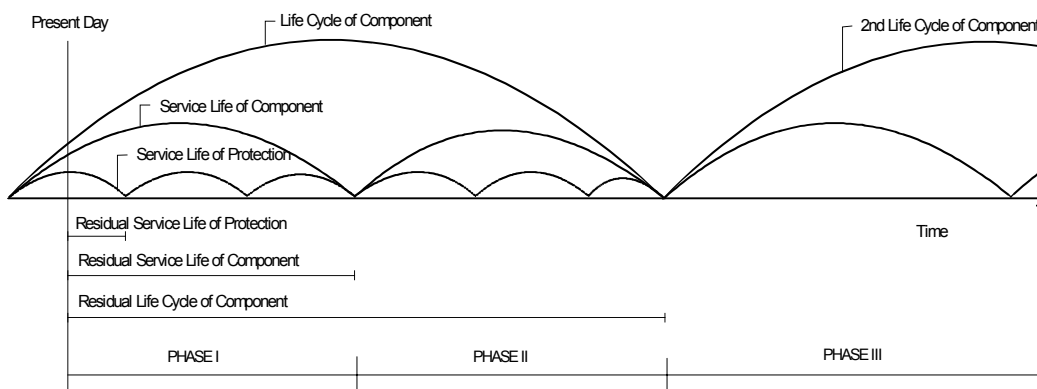


Figure 30. Definition of action for 3 phases in the lifetime of a concrete structure.

A special moment of time during the life cycle of a component is the present day. It is usually considered to be the same as the moment of design. Behind the present day there is the history of the component which may be known or unknown. However, it is assumed that the condition of the component and the condition of possible protections on it is known, as they are at the moment. Ahead of the present day there is the residual service life of the protection, the residual service life of the component (until the next repair) and the residual life cycle of the component (until the replacement of the component). After the on-going life cycle a new life cycle of the component can be started if desired.

A LC designer specifies the actions for the three phases as follows:

- Phase I All actions of protection and patching by which the residual service life is aimed to be extended are defined.
- Phase II The repair methods, criteria for the time of repair, number of possible consecutive repairs etc. are defined until the end of the life cycle of the component. The patching and protection methods for this period of time can be defined in another way than for the on-going service life. This is necessary as the need of protection may be changed after the repair.
- Phase III The methods of renovation, criteria for the time of renovation and the number of consecutive renovations are defined. For this period of time the repair methods can be newly defined as also the patching and protection methods. This is necessary as the needs of MR&R actions may be changed after the renovation of the component.

The definitions of actions are not obligatory at all phases. If only the rest of the on-going service life is the only interest to a designer, they can define only the actions of Phase I and leave the other phases without attention. If it is considered that the component will not be repaired after the first service life but be replaced by a new one, Phase II can be omitted. If the life of the component will probably never be extended by a new life cycle, Phase III may be left without attention.

3.3.1 Guiding Columns

The definitions of actions in the previous chapter are put into action through Guiding Columns for the Markov Chain LC Table. By the Guiding Columns the definition of MR&R methods used, the condition guarding and timing of actions and counting of the successive applications of MR&R actions are performed.

There are 8 guiding columns for each MR&R action category. The following information is given by them:

1. Is the MR&R action category defined for the phase (I, II or III)? If yes, a "1" sign is placed along with time for that phase.
2. Is there a forced timing for the action category? A forced timing may logically result from another MR&R action addressed to the same component. If yes, a "1" sign is placed at that year.
3. Timing of action. This column is the most important as it defines the timings of actions in the Markov Chain LC Table. It consists of a logical formula that compares the real probability of exceeding the limit state with the maximum allowable probability and the real number of actions of the same type with the maximum number. If the result of the logical formula is "True", a "1" sign is placed at that year.
4. MR&R system. The number placed in this column corresponds to the code number of the selected MR&R system in the Tables of MR&R systems.
5. Real probability of exceeding the limit state. The results of this column are linked from the Markov Chain LC tables where this probability is calculated from year to year.
6. The maximum allowable probability of exceeding the limit state. This is given by the designer in the definition of actions. The probability may be different in different phases of design, however.
7. Number of consecutive actions of the same type. The system automatically counts this number.
8. Maximum allowable number of consecutive actions of the same type. This is given by the designer in the definition of actions.

3.4 Determination of Life Cycle Costs

3.4.1 General

The last phase in the life cycle cost analysis is the determination of life cycle costs. The cost counters get their information from the Markov Chain life cycle table and the table of the MR&R systems. The task of the cost counters is to collect and summarise the costs from the total time frame e.g. design period. The costs that are determined are:

- MR&R costs
- user costs
- environmental impacts

The life cycle costs are counted based on ISO whole life costing principles [4]. As a result the costs are determined in two ways:

- real costs and
- present value costs.

The real costs refer to maintenance costs calculated using the current labour and material costs throughout the treated time (no discounting). The total real costs (per unit) are calculated from the equation:

$$C_R = \sum_{i=0}^t \sum_{j=1}^{n_i} C_{j;i} \quad (47)$$

where

- C_R is the sum of real costs from the treated time frame, Euro/m²,
 $C_{j;i}$ costs of the jth maintenance action in year i, Euro/m²,
 n_i number of maintenance action in year i, and
 t number of years in the time frame (length of the span in years).

Present value costs refer to maintenance costs discounted to the present day by the discount factor. As the discount factor diminishes with time, the PV costs of actions scheduled near to the start of the time frame are greater than the PV costs of respective actions scheduled later in the time frame. The present value costs are calculated from the equation:

$$C_{PV} = \sum_{i=0}^t \sum_{j=1}^{n_i} C_{j;i} \frac{1}{(1+r)^i} \quad (48)$$

where

- C_{PV} is sum of discounted (present value) costs from the treated time frame, Euro/m², and
 r rate of interest.

To compare different maintenance strategies (especially with different lengths of the time frame) it is advisable to redistribute the sum of LC costs evenly into annual costs. This can be done based either on real costs or PV costs, so two kinds of annual costs are defined:

- average annual costs and
- equalised annual costs

The average annual costs are defined as the sum of real costs divided by the number of years in the time frame:

$$A_A = \frac{C_R}{t} \quad (49)$$

where

A_A is average annual costs, Euro/m²/year.

The equalised annual costs are determined by multiplying the sum of discounted costs by the annuity factor:

$$A_E = C_{PV} \cdot \frac{r(1+r)^t}{(1+r)^t - 1} \quad (50)$$

where

A_E is equalised annual costs, Euro/m²/year.

The equalised annual costs depend on how the maintenance actions are scheduled within the time frame. Maintenance actions scheduled near to the start of the time frame increase the equalised annual costs more than those scheduled later in the time frame. This feature is emphasised with increasing rate of interest.

3.4.2 Determination of MR&R Costs

The MR&R costs are comprised of real maintainer costs such as costs of coating, protection, patching, repair, rehabilitation, renovation etc. They do not contain capital costs or disposal costs.

The unit costs of MR&R actions are usually based on statistical data from earlier executed MR&R projects. In some cases the costs depend on the extent of the repair i.e. the area of repair and the depth of concrete that is replaced from the structure. The unit costs may also be dependent on the general condition of the structure. Then a single value is not justified for unit costs. Instead a model formula that determines the unit costs as a function of the relevant parameters is created. An example of such a model formula is given in the following Equation 51:

$$UnitCost = UnitCost_0 \cdot C_{depth} \cdot C_{area} \cdot C_{cond} \quad (51)$$

where

Unit Cost is unit costs of a MR&R action, Euro/m²

UnitCost₀ unit cost of a MR&R action respective to the minimum depth and the minimum area of repair, Euro/m²

C_{depth} coefficient depending on the depth of repair

C_{area}	coefficient depending on the area of repair
C_{cond}	coefficient depending on the condition of the structure at the moment of repair

The coefficient C_{depth} is determined from the Equation 52:

$$C_{depth} = 1 + c_d \cdot \text{Min}\left(0; \frac{D - D_{min}}{D_{min}}\right) \quad (52)$$

where

c_d	is	coefficient
D		depth of repair, mm
D_{min}		minimum depth of repair, mm

The area coefficient C_{area} is determined from the Equation 53:

$$C_{area} = z + \left(1 - \frac{\text{Min}(A_{max}; A) - A_{min}}{A_{max} - A_{min}}\right) \cdot (1 - z) \quad (53)$$

where

z	is	relative proportion of the costs ($0 \leq z \leq 1$) corresponding to the maximum repair area
A		repair area, m^2
A_{max}		maximum limit for the range of area in which the costs depend on the repair area, m^2
A_{min}		minimum limit for the range of area in which the costs depend on the repair area, m^2

The unit costs of a MR&R action may also depend on the condition of the structure at the moment of repair. The coefficient of condition may be determined as in Equation 54:

$$C_{cond} = (1 + f_{LS})^E \quad (54)$$

where

f	is	the fraction of the component exceeding the limit state
E		exponent (smaller or greater than 1)

The unit maintenance costs and the coefficients C_{depth} and C_{area} are determined in the tables of MR&R systems. The coefficient C_{cond} is determined in the Markov Chain life cycle table after determination of the fraction exceeding the limit state.

Example

The unit costs of a MR&R action is 100 Euro/ m^2 for the minimum depth of repair 10 mm and for the maximum area of 60 m^2 . The real depth of the repair is 40 mm and coefficient c_d is 0.14. The real area of repair is 40 m^2 and the minimum area in the range in which the unit costs vary is 10 m^2 . The value of relative proportion of the cost z is 0.7. The condition of the structure at the

moment of repair is such that about 40 % of the structure exceed the limit state and value of exponent E is evaluated as 0.9.

The coefficients are determined as follows:

$$C_{depth} = 1 + 0.14 \cdot \frac{40mm - 10mm}{10mm} = 1.42$$

$$C_{area} = 0.7 + \frac{35m^2 - 10m^2}{60m^2 - 10m^2} (1 - 0.7) = 0.85$$

$$C_{cond} = (1 + 0.4)^{0.9} = 1.35$$

So the evaluated unit costs are:

$$UnitCosts = 100 \cdot 1.42 \cdot 0.85 \cdot 1.35 = 163 \text{ Euro} / m^2$$

3.4.3 User Costs

In some types of infrastructure, such as bridges, the user costs are included in the decision making on maintenance strategy. For bridges three kinds of road user costs (RUC) can be identified [6]:

- additional road user costs due to restricted traffic for restricted axle loads and inadequate bridge geometry
- additional road user costs due to maintenance works (delays)
- risk costs due to failure of a bridge.

The risk costs are not treated in this context.

Additional User Costs due to Restricted Traffic

The additional road user costs due to restricted traffic can be divided into the following two categories:

- Driving depending costs (DC) and
- Time depending costs (TC)

These can be evaluated using the following general Equations 55 and 56 [7]:

$$\text{Driving depending cost} \quad DC = V_{truck} \cdot C_D \cdot D \quad (55)$$

$$\text{Time depending costs} \quad TC = V_{truck} \cdot C_T \cdot \frac{D}{S} \quad (56)$$

The notations in Equations 55 and 56 are:

V_d	truck traffic on the diversion route because of the restriction, trucks/d
C_D	average driving costs per unit, Euro/km

C_T	average time costs per unit, Euro/h
D	length of the diversion, km
S	speed of truck traffic on the diversion route, km/h.

The Equations 55 and 56 apply to any restriction of the traffic. Whether it is due to inadequate load bearing capacity or to inadequate dimensions of the bridge they are applicable.

The user costs determined by the above-presented equations are addressed to the whole bridge. The problem related to component based life cycle cost analysis is how to divide the user costs to the components.

Additional User Costs due to Maintenance Works

User costs due to maintenance works form a major part of the user costs for bridges. This kind of user costs result from the following reasons:

- reduced speed
- diversion
- signal regulation

The road user costs for maintenance works can be determined using the following formulas in the Table 3.

Table 3. Formulas of the road user costs [6]

Reduced speed	$RUC = I \cdot \%_{car} \cdot \left(\frac{L}{v_{red;car}} - \frac{L}{v_{org;car}} \right) \cdot TDC_{car} +$ $I \cdot \%_{truck} \cdot \left(\frac{L}{v_{red;truck}} - \frac{L}{v_{org;truck}} \right) \cdot TDC_{truck}$	(57)
Diversion TC is time dependent costs DC is driving dependent costs	$RUC = TC + DC$ $TC = I \cdot \%_{car} \cdot \left(\frac{L_{div}}{v_{div;car}} - \frac{L_{org}}{v_{org;car}} \right) \cdot TDC_{car} +$ $I \cdot \%_{truck} \cdot \left(\frac{L_{div}}{v_{div;truck}} - \frac{L_{org}}{v_{org;truck}} \right) \cdot TDC_{truck}$ $DC = I \cdot \%_{car} \cdot (L_{div} - L_{org}) \cdot DDC_{car} +$ $I \cdot \%_{truck} \cdot (L_{div} - L_{org}) \cdot DDC_{truck}$	(58)
Signal regulation F _{avr} is average delay per vehicle	$RUC = I \cdot \%_{car} \cdot \frac{F_{avr}}{3600} \cdot TDC_{car} +$ $I \cdot \%_{truck} \cdot \frac{F_{avr}}{3600} \cdot TDC_{truck}$	(59)

The notations in the equations are the following:

I	average daily traffic (ADT)
% _{car} , % _{truck}	percentage of traffic for cars and trucks
L	length of the affected route, km
V _{org} , V _{red}	average original and reduced speed, km/h
TDC _{car} , TDC _{truck}	time dependent unit costs for cars and trucks, Euro/h
L _{div} , L _{org}	length of the diversion and the original route, km
V _{div} , V _{org}	average speed on the diversion and the original route, km/h
DDC _{car} , DDC _{truck}	driving dependent unit costs for cars and trucks, Euro/km
F _{avr}	average delay per vehicle for the signal regulation, s

The delay per vehicle for signal regulation is evaluated by the Equation 60:

$$F_{avr} = \frac{r^2}{2 \cdot C \cdot (1 - b \cdot I)} \quad (60)$$

where

r is road time, s
 b average handling time per vehicle (normally 2s/PCU)
 I traffic intensity per direction
 C rotation time

The above-presented equations refer to the user costs per day. So the total road user costs depend on the delay time i.e. the time of the repair. The total costs per functional unit like m², m etc. can be determined as the product of the user costs per day and the repair time.

The road user costs can be assigned to components of bridges by evaluating the repair time of components. The repair time may be evaluated based on the production rate of the work [m²/day] for each MR&R method and the area of repair. Then the time of repair is determined as in Equation 61:

$$t_r = \frac{A}{a_r} \quad (61)$$

where

t_r is repair time, d
 A area of repair, m²
 a_r production rate of the MR&R method applied, m²/d.

This calculation method is not indisputable as in practice several works for several components can be performed at the same time. However, this offers one solution for the problem of addressing user costs for components.

3.4.4 Environmental Impacts

The purpose of the environmental impact analysis in LIFECON is to provide the decision-makers with comparative data on the environmental impacts of various MR&R methods and life cycle action profiles. This data is used as one attribute in the optimisation of the maintenance strategy for structures.

As a starting point of the environmental impact analysis for the LCAPs it is assumed that the environmental profiles for the used materials are available. The profiles should at least consist of the following variables:

Resources of energy [MJ]

- Renewable energy
- Non-renewable energy

Emissions into air [kg], [g] or [mg]

- CO₂
- SO₂
- NO_x

- Particles
- CH₄
- Non-methane VOC

Non-renewable raw materials [kg]

- mineral raw materials

In a situation when environmental analyses are not available specifically for each product, estimations for environmental profiles are determined for products of the same material group as follows:

- Epoxy coating
- Acrylic coating
- PUR coating
- Cement based coating
- Polymer-cement coating
- Wax
- Silane impregnation
- Acrylic impregnation
- Teflon impregnation
- Cement based mortar layer
- Shotcrete layer
- Cathodic protection
- Mechanical repair by shotcrete
- Mechanical repair by casting
- Patching with mortar
- Realkalisation
- Electrochemical removal of chlorides
- New construction
- etc.

The results of the environmental profiles are normally given per mass units, [kg]. So the profiles must first be converted into functional units, usually [m²]. To do this each environmental variable is divided by the coverage [m²/kg] of the material. So the environmental profiles of materials in functional units depend on the thickness of the material layer.

The emissions related to MR&R actions have many kinds of ecological impacts. The following classification of impacts is normally used:

- Climate change
- Acidification
- Formation of photo-chemical ozone
- Ecotoxicity
- Heavy metals
- Cancerous materials
- Effect in biodiversity

The first three environmental impact classes have usually been applied in the analyses of the construction sector.

The environmental impact analysis continues now by characterisation of the environmental variables with respect to each environmental impact class. This means multiplication of environmental variables with a characterisation coefficient, which takes into account the relative importance of the variable for the impact in each environmental impact class. Next the impact classes are normalised by a normalisation factor and weighted by a weighting factor so that after all the total environmental impact can be described only by one overall indicator.

There are several methods developed for the evaluation of the total environmental impact. However, there is no general agreement on the methods as yet. In Table 4 the internationally most usual Life Cycle Impact Analysis procedures (LCIA) and their features are presented.

Table 4. Summary of classification, characterisation, normalisation and weighting approaches [8].

LCIA Procedure	Classification and Characterisation	Normalisation and Weighting
CML Leiden University, the Netherlands	Classification Relating to issues concern from a European and global perspective Characterisation Fate and relative environmental intervention e.g. exposure or depletion modelling (European continent except for global interventions) or an inventory constituent compared to a specific substance or parameter	Normalisation Choice of normalised values given for: – World population (1990) – The Netherlands (1997) – Western Europe (1995) Weighting No weighting procedure included or recommended
Ecopoints BUWAL, Switzerland	Classification Relating to issues of concern from a Swiss and global perspective Characterisation Fate and relative environmental intervention e.g. exposure or depletion modelling (Switzerland and European continent except for global interventions) or an inventory constituent compared to a specific substance or parameter	Normalisation The target/critical inventory (mass or energy) for each impact category for Switzerland over one year. The target/critical inventory refers to the aimed level set by the Swiss authorities. Weighting Calculated as the ratio of actual inventory value to the target/critical value for each impact category
Eco-indicator 95 Pré Consultants, the Netherlands	Classification Relating to issues of concern from a Netherlands and global perspective Characterisation Fate and relative environmental intervention e.g. exposure or depletion modelling (the Netherlands and European continent except for global interventions) or an inventory constituent compared to a specific substance.	Normalisation Normalisation is based on 1990 effect levels for Europe excluding the former USSR. Weighting Calculated as the ratio of actual inventory value to the target/critical inventory value for each effect category, with additional subjective weighting to represent significance on human health and ecosystem impairment from a Netherlands perspective.

(continue)

Table 4. Continue.

LCIA Procedure	Classification and Characterisation	Normalisation and Weighting
Eco-indicator 99 Pré Consultants, the Netherlands	Classification Relating to human health, ecosystem and abiotic resource damage concerns from Netherlands and global perspective. Characterisation Actual damage modelling (the Netherlands and European continent except for global interventions) of an inventory constituent on human health (Europe), ecosystems (the Netherlands) and mineral and energy resources (global)	Normalisation Total inventory of mass and energy used (mostly for 1993 as base year) for the whole of western Europe for one year per person (population of 495 million assumed) Weighting A choice of four based on responses from a panel of European scientific experts placed into three perspectives: <ul style="list-style-type: none"> – Individualists (higher weight to human health) – Egalitarians (higher weight to ecosystem quality) – Hierarchists (equal weight distribution)
EPS Chalmers University, Sweden	Classification Relating to human health, biotic and abiotic resource damage concerns from a Swedish and global perspective. Characterisation Actual damage modelling (Sweden and European continent except for global interventions) of an inventory constituent on human health, biotic and abiotic resources.	Normalisation No formal normalisation introduced into method Weighting Calculated as the willingness to pay (WTP) to restore impacts, where the environmental reference is the present state of the environment (Swedish perspective). Overall indicator is the Environmental Load Unit (ELU).

More information of life cycle impact analyses is given in LIFECON Deliverable 5.3, Methodology and Data for Calculation of Life Cycle Costs of Maintenance and Repair Methods and Works.

In the following only one method, the Swedish Environmental Priority Strategy (EPS) -method is presented [9]. The overall indicator in this method is Environmental Load Unit (ELU) which is defined in Euro as follows:

$$\begin{aligned}
 ELU[\text{Euro}] = & 1.557 \cdot CH_4 [\text{kg}] + 0.191 \cdot CO [\text{kg}] + 0.0635 \cdot CO_2 [\text{kg}] \\
 & + 0.00707 \cdot \text{particles} [\text{kg}] + 3.4 \cdot (C_xH_y - CH_4) [\text{kg}] + 0.395 \cdot NO_x [\text{kg}] \\
 & + 0.0545 \cdot SO_x [\text{kg}]
 \end{aligned}
 \tag{62}$$

The principle for calculating the sum of environmental impacts from MR&R actions during the treated time frame is the same as that for life cycle costs. The unit environmental impacts per functional units like m², m etc. for each MR&R action are determined in the MR&R system tables. Using these unit costs the total costs for the whole treated time frame are determined in the counter columns of the life cycle calculation table by adding the impacts each year when the MR&R actions are triggered.

3.5 Results of LCC Analysis

3.5.1 Direct results of LCC analysis

The main results of LCC analysis can be compacted into a small table of results. Table 5 shows the LC costs calculated per unit area. The annual unit costs are calculated as average annual costs and equalised annual costs (ref. Chapter 3.5.1).

Table 5. Results of LCC analysis. Unit Costs

Unit Costs	MR&R Costs
Cumulative Real Costs, Euro/m ²	2114
Cumulative PV Costs, Euro/m ²	98
Average Annual Costs, Euro/m ² /year	8.46
Equalised Annual Costs, Euro/m ² /year	3.91

The true component costs are obtained by multiplying the unit cost by the surface area of the component. Supposing, for example, that the surface area of the component is 166 m², then the true costs are those presented in Table 6.

Table 6. Results of LCC analysis. True Component Costs.

Unit Costs	MR&R Costs
Cumulative Real Costs, Euro	350905
Cumulative PV Costs, Euro	16235
Average Annual Costs, Euro/year	1404
Equalised Annual Costs, Euro/year	649

The design period was in this case 250 years. The condition of the structure changes during this time as depicted in Figures 31 and 32.

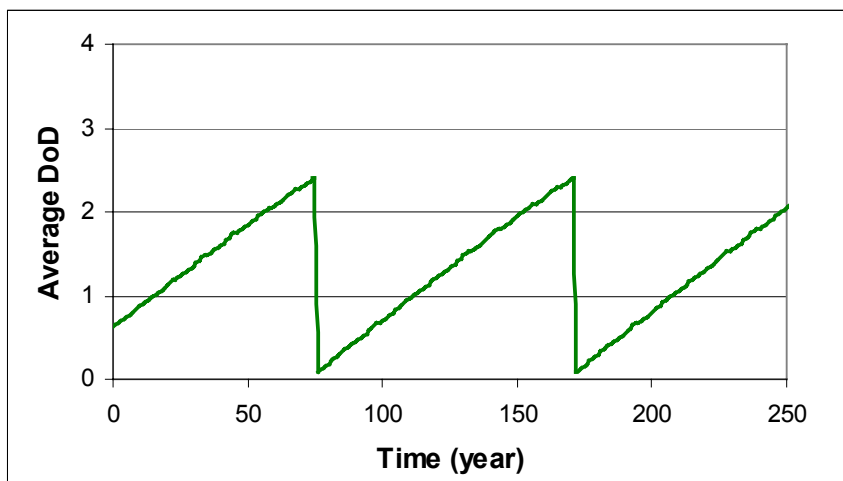


Figure 31. Average Degree of Damage as a function of time.

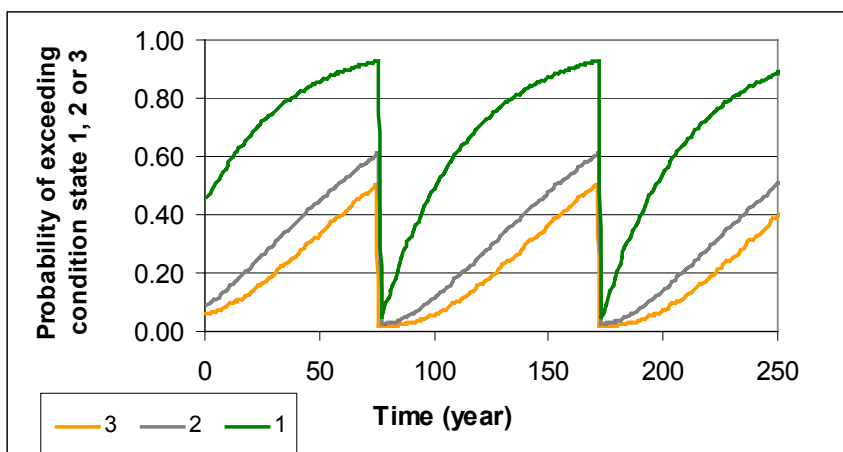


Figure 32. Probability of exceeding the condition state 1, 2 and 3 as a function of time.

In this example the maximum allowable probability of exceeding the condition state 3 (= limit state) was 50 %. Comparing Figure 32 and Figure 31, it is seen that the repair was triggered immediately every time when this limit was exceeded.

The costs can also be presented as a function time. Figure 33 shows the cumulative MR&R costs per unit area as real costs and PV costs. The MR&R costs in this case were composed of structural repair cost and coating costs.

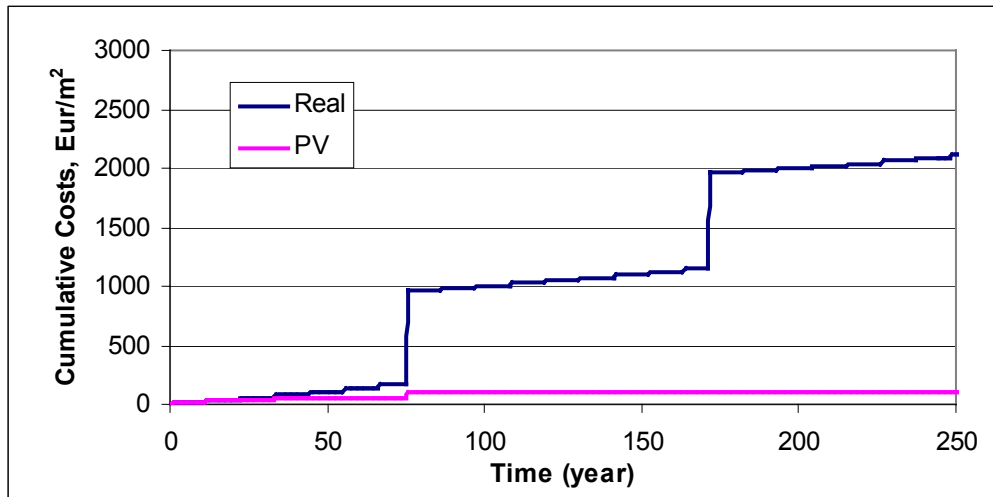


Figure 33. MR&R costs per unit area presented cumulatively as a function of time.

3.5.2 Comparison of Life Cycle Action Profiles by LCC Analysis Results

The following example presents some LCC analysis results for an edge beam of a bridge. Edge beams are usually exposed to rain and chloride-contaminated splash water from the bridge deck. Because of the great moisture and chloride stresses, the degradation rate of both the structural concrete and the possible coatings on the beam is rapid. On the other hand the coatings may have a great effect on the degradation rate of the structural concrete. The treated time frame in these examples was 250 years and the rate of interest 4 % (for equalised costs).

In Table 5 the coatings are assumed to be reapplied every time the limit state of coatings (3) is exceeded with the maximum allowable probability and the edge beam is assumed to be structurally repaired every time the limit state of the structure (3) is exceeded with the maximum allowable probability of 50 %. The analysis results shows that the average annual costs of edge beams are approximately 6 - 8 Euro/m/year without coatings. However the use of some coatings, such as silane-based impregnation agents or PUR-water membranes, may be cost effective. The time for the reapplication may be a little delayed rather than advanced from the estimated service life of the coating.

Table 7. Average and equalised annual costs (Euro/m/year) of an edge beam with alternative coatings. High moisture and chloride stress.

Coating	Average annual costs Euro/m/year			Equalised annual costs Euro/m/year		
	Probability of exceeding condition state 3 for coatings			Probability of exceeding condition state 3 for coatings		
	20%	50%	80%	20%	50%	80%
No coating	6.95	6.95	6.95	8.08	8.08	8.08
Acryl coating	10.83	10.52	9.58	13.01	12.12	11.51
Epoxy coating	10.52	10.31	9.58	12.07	11.14	10.47
Copolymer coating	10.49	9.25	9.58	12.09	10.49	9.50
Cement based coating	14.02	12.93	11.44	15.51	14.86	12.61
Silane impregnation	6.57	6.28	5.49	5.26	5.09	5.59
PUR-water membrane	8.11	6.46	4.81	7.37	4.84	4.19
Shotcrete 30 mm	20.41	12.63	9.70	25.20	11.77	7.83

In Table 6 the optional repair methods of an edge beam are compared. The edge beam is assumed to be repaired every time when the limit condition state of the beam is exceeded with the maximum allowable probability. The analysis results show that the repair by casting concrete is more cost effective than shotcrete repair. However, the electrochemical removal of chlorides may still be more cost effective whenever it is possible to use this method. In the cases of shotcrete repair and electrochemical removal of chlorides, it was assumed that the maximum allowable probability could not be higher than 30 % because of excessive disintegration of concrete.

Table 8. Average and equalised annual costs of an edge beam (Euro/m/year) with alternative repair methods. High moisture and chloride stress.

Repair method	Average annual costs Euro/m/year			Equalised annual costs Euro/m/year		
	Probability of exceeding condition state 3 of the structure			Probability of exceeding condition state 3 of the structure		
	10%	30%	50%	10%	30%	50%
Shotcrete	18.50	12.33	(9.87)	27.73	13.14	(7.57)
Repair by casting concrete	13.90	8.69	6.95	24.30	12.90	8.08
Electrochemical removal of chlorides	7.84	4.48	(3.36)	10.66	5.26	(3.07)

The purpose of the above presented LCC analysis results is only to demonstrate the possibilities of a LCC analysis and show how the analysis results could be interpreted to get answers to strategical maintenance questions of bridge owners. The results of this example are not meant for making conclusions on the real maintenance policy of edge beams as the results are highly dependent on the initial data used. The initial data include the data on unit costs, rate of degradation and protective properties of coatings as well as data on unit costs, rate of degradation and action effects of repair methods.

Figure 34 shows how the maximum allowable probability affects the average condition state and the frequency of repairs of a structure. By decreasing the maximum allowable probability the average condition state is clearly decreased. However, the frequency of repairs is increased and the LC costs are probably also increased by lowering the maximum allowable probability.

The maximum allowable probability must be evaluated taking into account the demands of safety, structural performance, convenience in use and aesthetic appearance. These demands form the edge conditions of an LCC analysis problem. If for instance the users of the structure demand that the maximum allowable probability must be no more than 10%, then higher LC costs must be accepted than those corresponding to higher levels of maximum allowable probability. On the other hand if there is not enough money to keep such a low level of maximum allowable probability then a little lower standard for safety, convenience in use and aesthetic appearance of structures must be accepted.

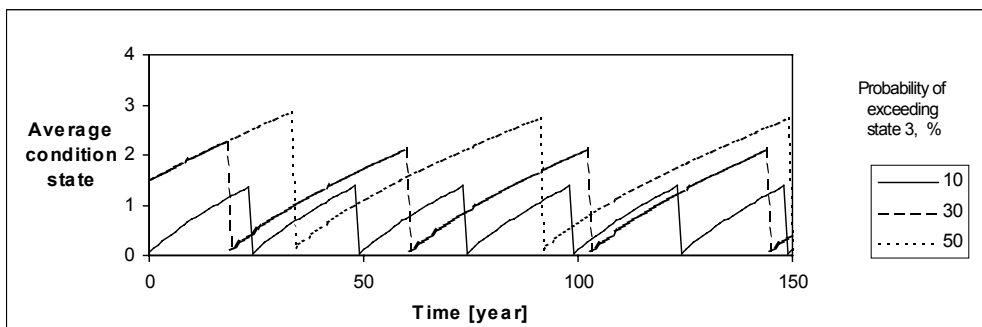


Figure 34. The effect of the maximum allowable probability on the average condition state and the frequency of repairs of a structure.

3.6 Types of LCC Analysis in Lifecon LMS

The Markov Chain based LCC analysis is used in LIFECON LMS in the following modifications

- Automatic / Manual LCC analysis
- Object Level / Network Level LCC analysis
- Surface Damage / Structural Damage LCC analysis

3.6.1 Automatic / Manual LCC analysis

Automatic LCC analysis refers to an analysis type in which the definition and timing of MR&R actions is performed automatically using the decision tree method and the condition controlled Markov Chain LC table. In a manual analysis the definition and timing of MR&R actions is done manually. However, even in the manual analysis the automatic methods are available.

The following automatic routines are available in both the automatic and manual analysis systems:

- automatic application of object and component specific parameter data for degradation, action effect and cost models,
- automatic conversion of degradation models into Markov Chain transition probabilities,
- automatic arrangement of the guiding columns and determination of the annual condition state distributions according to the specifications of the user,
- automatic determination of the annual condition state distributions in the Markov Chain LC tables,
- automatic calculation of LC costs, user costs and environmental impacts.

Figure 35 shows an example of a decision tree. A decision tree has a "root" which forks at "nodes" representing the relevant criteria (related to e.g. material, structural and environmental conditions) and makes with a growing number of nodes an ever-increasing amount of "branches". The final branches after the last nodes are called "leaves". The optimum LCAPs are the results of the tree and are inserted in the leaves of the tree.

Choose the component:

Component

- Main girder, beam
- main girder, vault
- Main girder, box beam
- Internal surfaces of boxbeam
- Secondary girder
- Pylon 0-3 m**

Use component specific values
 Use default values of the component

Data of component

Thickness,	500	Target service life, years	100
Width, m	2	Cement	SR
Length, m	3	Nomial strength, MPa	39.5
Surcace area, m ²	24	Air content, %	3
Direction of surface	P	Concrete cover, mm	32
Moisture stress	0.8		
Chloride stress (road)	0.9		
Chloride stress (cross road)	0		

Default values by red colour

Using the Decision tree method:

Do a LCC plan for component

Manually:

Change the definition of actions

Change the timing of actions

Return

Results

Total MR&R costs	22570	LCC results
Residual service life	158	LCA results

Figure 36. Form for Manual LC design in a Markov Chain based LCC analysis programme.

The designer can change the automatically prepared plan manually in three ways:

- by changing the component specific data (by changing the values in the component form)
- by changing the definitions of MR&R actions (by pressing the "definition of actions" button to open another form is opened) and
- by changing the timing of actions (by pressing the "timing of actions" button to open another form)

In the form for "timing of actions" the timings of the automatic design can be seen as dots in the design table. Using the form a designer can remove actions or add them or change the timing arbitrarily. The design table can also be filled by repeated actions with regular intervals.

3.6.2 Object Level / Network Level LCC analysis

The object level and the network level analysis methods differ in the input data used, in some details of the calculation procedure and in the output of results.

An object level analysis usually applies object and component specific data in degradation, action effect and cost models. A network level analysis uses default (average) values of the whole network of the objects or average values calculated from some subnetwork of the objects.

The main difference in the calculation procedure of the network level analysis method as compared to the object level procedure is in the response of exceeding the maximum allowable probability for the limit state. In an object level analysis, exceeding the maximum allowable probability triggers an action for the whole component in need of repair while in the network level analysis only the fraction of the structural network which overrides the maximum allowable probability is repaired.

The output of an object level analysis consists of documentation of preliminary project plans. The output of a network level analysis consists of reports on technical and financial strategic planning and scenarios on future MR&R needs and costs.

Figure 37 shows an example of a network level long-term analysis (illustrative presentation). In this case there was a criteria set for the condition state 4 that no more than 15 % of structures were allowed to drop to condition state 4 without repair. The maximum annual budget was defined to be 36500 Euro.

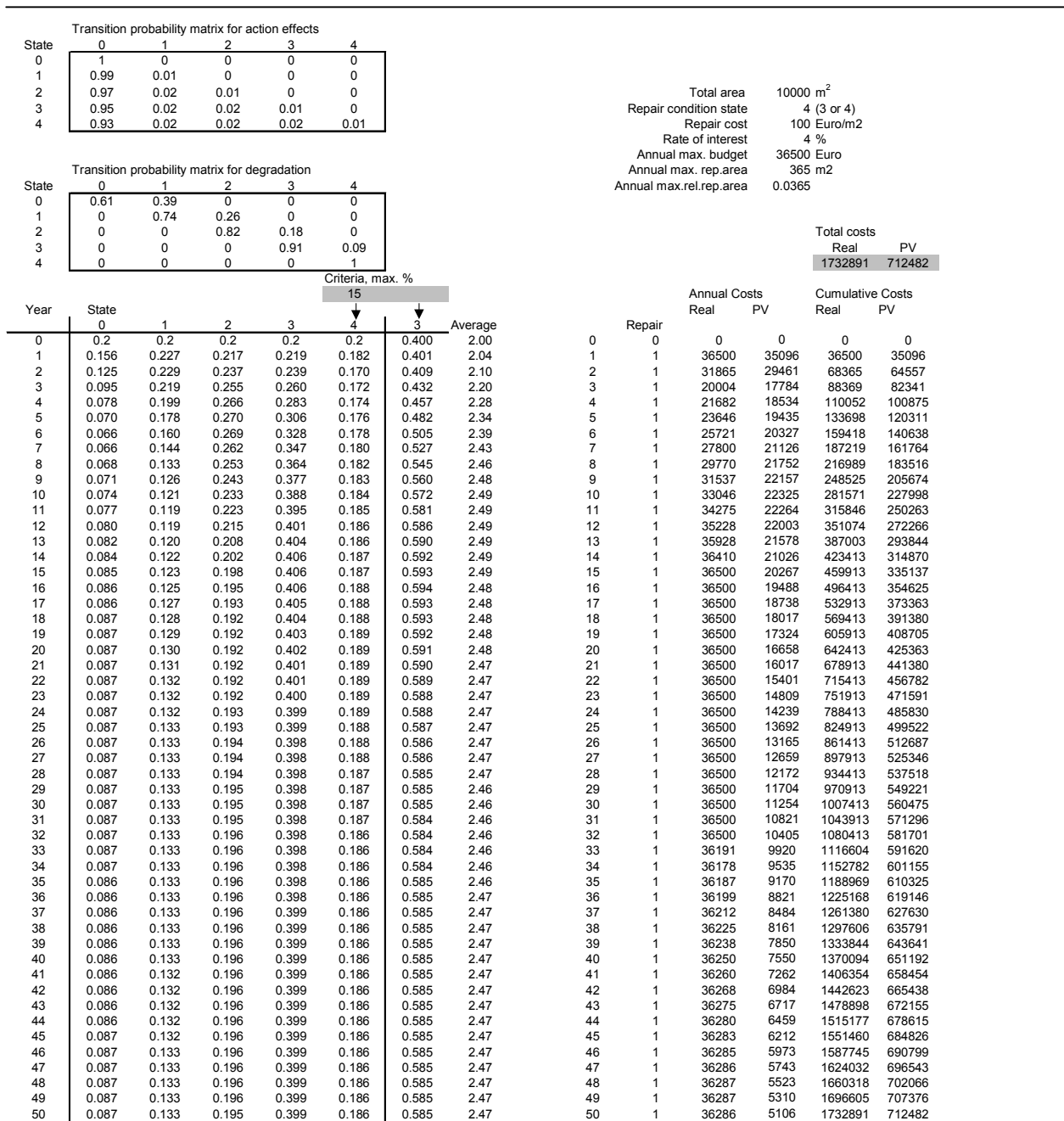


Figure 37. Long-term optimisation with a Markov Chain LC analysis table.

As seen from the results, the condition state distribution approaches a steady state distribution. After some 30 years the distribution does not change much any more. In this case the initial condition state distribution was uniform, i.e. 0.2 in each condition state i ($i=0,1,2,3,4$). However, the final condition state distribution is independent of the initial condition state distribution. The final distribution can be considered as representing the long-term optimum.

3.6.3 Surface Damage / Structural Damage LCC analysis

The surface damage and structural damage LCC analyses are based on different degradation modes. A surface damage LCC analysis uses surface damage degradation models and the

MR&R action sortiment is addressed to protection and repair surface damage. The surface damage analysis takes into account the most common degradation types such as carbonation, chloride penetration, corrosion of reinforcement and frost attack.

A structural damage LCC analysis also takes into account the above-mentioned degradation types but it focuses to the degradation process at a crack. In a bent structure the capacity of the structure is considered to be diminished by the corrosion of reinforcement at a crack and also by the degradation of concrete on the compressive side of the structure.

The surface damage analysis and the structural damage analysis can be treated as separate analyses but they can also be combined. In a combined analysis system both the analyses are conducted in parallel so that the repair needs for both surface repair and structural repair can be evaluated at the same time. This makes the analysis more complex but on the other hand it is reasonable as the surface repair actions and the structural repair actions may have interaction.

Both the surface damage and structural damage analyses are used in the object level LMS for project planning. Both analysis types are also applied in the network level LMS.

The rate of surface damage can be evaluated by the Reference structure models presented in the second part of this Deliverable. The principles of integrating these models into a Markov Chain based LCC analysis are presented in Chapter 2. The formulation of the structural damage models is briefly outlined in the following subsections.

Modelling of the Structural Damage

The decreasing load bearing capacity of a structure as a result of degradation in materials is evaluated by the help of the residual capacity factor (RCF). This factor expresses the load bearing capacity of a damaged structure in relation to its original (undamaged) load bearing capacity.

$$RCF = \frac{R'(t)}{R} \quad (63)$$

where

RCF is residual capacity factor,
 R original capacity of a structure (as undamaged), and
 R'(t) load bearing capacity of the structure as damaged.

The residual capacity factor of a damaged structure is evaluated by inserting the evaluated reduced cross-sectional area and reduced strength of materials (as a result of degradation) to the normal dimensioning equation of a structure and by thus determining the reduced load bearing capacity in relation to the original load bearing capacity. Also it is possible to insert time-related models for material strength and dimensions of structures. This enables prediction of the residual capacity factor with time.

The ultimate criterion for load bearing capacity (and structural service life) can be derived from a safety analysis. The residual marginal factor (RMF) can be defined as follows:

$$RMF = \frac{R'-S'}{R-S'} \quad (64)$$

where

RMF is residual marginal factor,
 S original loading requirement, and
 S' loading requirement at the time of treatment.

The minimum value for the residual capacity factor can be determined as derived from Equation 64:

$$RCF_{\min} = RMF_{\min} \left(1 - \frac{S}{R} \right) + \frac{S'}{R} \quad (65)$$

The RMF_{\min} is evaluated based on the safety requirement in each case. Usually it is not allowed that more than 10% of the original safety index (β) is lost. Thus the RMF_{\min} is about 0.9. The quantity R/S is the total safety factor and is determined with characteristic values of S and R . The principle of the determination of the structural service life based on RCF is presented in Figure 38.

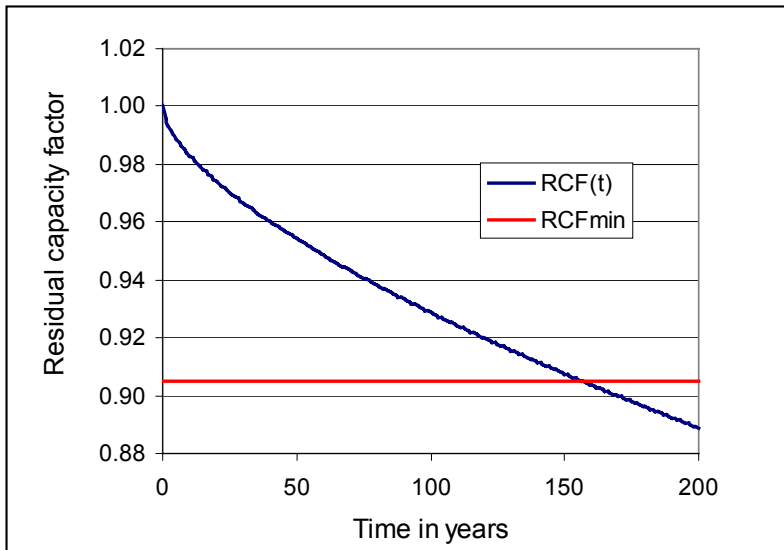


Figure 38. RCF-method for determination of the residual service life of structures.

In the case of bent reinforced concrete structures the residual capacity factor can be determined as follows:

$$RCF(t) = \frac{D'(t)}{D} \cdot \frac{z'(t)}{z} \cdot \frac{b'(t)}{b} \quad (66)$$

where

D is the diameter of the tensile steel rods,
 z the internal shaft of momentum of the structure, and
 b the breadth of the structure.

Equation 54 can be further presented as:

$$RCF(t) = \left(1 - \frac{2 \cdot s_s(t)}{D}\right) \cdot \left(1 - \frac{s_{cz}(t)}{z}\right) \cdot \left(1 - \frac{2 \cdot s_{cb}(t)}{b}\right) \quad (67)$$

where

- s_s is depth of corrosion at a crack (on the tensile side), and
 s_{cz} depth of degradation of concrete on top of the the compressive side of the structure, and
 s_{cb} depth of degradation of concrete on the vertical sides of the structure.

In the Equations 66 and 67 it is assumed that the corrosion at crack occurs only on one side of the steel rod (not all around) and that degradation of concrete can occur on all sides of the structure.

From Equation 67 one can see that the degradation model for corrosion at a crack can be inserted into the model of structural degradation. Also the model of degradation of concrete (on the compressive side) can be inserted in the model function. By dividing Equation 65 by Equation 67 a model function for structural degradation which is consistent with Equation 5 is obtained. So the problem of structural degradation can be solved using the same methods as those used with problems of surface damage. Accordingly, also the Markov Chain based LCC analysis can be used for structural studies by applying the above-presented principles.

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PART 2:
REFERENCE STRUCTURE MODELS FOR PREDICTION
OF DEGRADATION

1 Introduction

The Reference structure models were originally created in 1999 at VTT Building and Transport as a result of a research project financed by the Finnish Road Administration (FinnRA). These models were planned to be utilised for evaluating the rate of degradation and need of repair in the processes of the Finnish Bridge Management System. During the LIFECON project the models were further developed and systemised. They can be used in LIFECON LMS in LCC analyses at both network the and object level planning and in service life design. Another possible set of degradation models, "Duracrete models", is presented in LIFECON Deliverable 3.2.

The Reference structure models cover the most important degradation types of concrete structures: carbonation, chloride contamination, corrosion of reinforcement and the two types of frost damage: frost-salt scaling and internal frost damage. As corrosion of reinforcement may occur generally on all surfaces of a structure or locally at cracks, two types of models were created: 1) surface damage (physical and chemical degradation of concrete surfaces and general corrosion of reinforcement) and 2) crack damage (physical and chemical degradation of concrete at cracks and corrosion of reinforcement at cracks). The input data include parameters of environmental loads, structural details and material properties.

2 Basis for the development of degradation models

2.1 General Scheme

The rate of degradation is highly dependent on the environmental conditions. Of significance is not only the macro climate (resulting from the meteorological conditions of the building site) affect the rate of degradation but also the micro climate (resulting from the performance and condition of the drainage system of the building, condition of joints, protective effects of neighbouring structures etc.).

To be able to evaluate the true rate of degradation for specific structural parts the degradation models were developed using the scheme presented in Figure 1. In principle the same system can be applied to any degradation type.

In the first phase "laboratory models" which are applicable in constant environmental conditions were developed. These models are based on theoretical examinations and laboratory tests. In some models, such as those for carbonation and chloride penetration, laws of diffusion can be applied. By the laboratory tests the basic data of the effects of various material parameters on the rate of degradation can be studied. Also the effects of environmental parameters, such as moisture content and temperature, can be studied by laboratory tests.

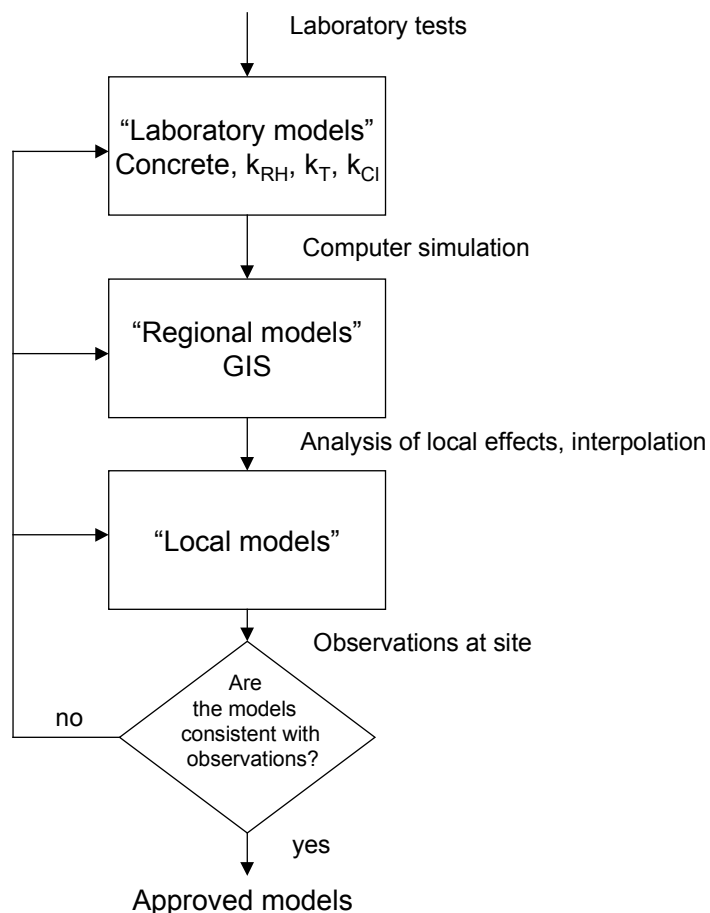


Figure 1. Development phases of durability models.

In the second phase "regional models" for natural weather conditions were developed. In natural weather conditions both the temperature and the moisture content of concrete change all the time. So the laboratory models that were created for constant conditions are not applicable as such in natural conditions. However, they can serve as initial data in the calculation processes which result in regional models. By dividing the time into short discrete steps and assuming that constant conditions prevail during each time step, it is possible to determine the incremental degradation corresponding to the time steps. Then by integrating the incremental degradations the total degradation in regional atmospheric conditions can be determined. In practice this process is connected to computer simulation by which it is possible to determine the momentary temperature and moisture contents of concrete in a model structure which is exposed to normal weather (reproduced by weather models).

The third phase was for development of "local models", i.e. evaluating the local (micro climatic) effects on the rate of degradation. An interpolation method based on the "moisture index and the "chloride index" was created for this purpose. "Moisture index" and "chloride index" are parameters which express the degree of moisture stress and the degree of chloride stress to which a specific structural part is exposed. By first determining the rate of degradation for "basic cases", i.e. for moisture index 0 and 1 and for chloride index 0 and 1, the intermittent values for degradation rate can be evaluated by interpolation (by applying the moisture index and chloride index values).

2.2 Laboratory Models for Constant Conditions

2.2.1 Chemical Changes in Concrete (Carbonation and Chloride Penetration)

The protection provided by concrete against steel corrosion is based on the alkalinity of concrete. In an alkaline environment a tight oxide layer is produced around the steel, preventing further corrosion. The pH of pore water in concrete containing solid calcium hydroxide is always at least 12.5, which is adequate to passivate steel. However, normally the pH of pore water is even higher, about 13 - 14, in alkali hydroxides containing cement pastes. When concrete **carbonates** its pH is reduced to about 9. Then the passive layer around the steel is destroyed and corrosion is initiated.

The fact that the depth of carbonation is approximately proportional to the square root of time can be theoretically derived by applying the diffusion theory with a 'moving boundary'. In this theory the carbon dioxide is diffused into concrete and reacts with the non-carbonated calcium minerals at the 'moving boundary', that is at the distance of X_c (depth of carbonation) from the surface of a structure. The carbon dioxide content between the surface and the moving boundary is assumed to be linear. Then the flux of carbon dioxide towards the moving boundary can be evaluated as [6]:

$$J = D \frac{\Delta C}{X_{carb}} \quad (1)$$

where

- J is flux of carbon dioxide, $\text{g}/\text{m}^2\text{s}$,
 D the diffusion coefficient of concrete with respect to CO_2 , m^2/s ,
 X_{carb} distance of the moving carbonation boundary from the surface of the structure, m ,
 $\Delta C_s = C_s - C_{xc}$, kgCO_2/m^3 ,
 C_s CO_2 content of air at the surface of concrete, kgCO_2/m^3 , and
 C_{xc} CO_2 content of air at the moving boundary, kgCO_2/m^3 .

The carbon dioxide flux into concrete must be in balance with the rate of reaction occurring at the boundary. The rate of reaction depends on the amount of non-carbonated calcium in concrete:

$$J_R = a \frac{dX_{\text{carb}}}{dt} \quad (2)$$

where

- J_R is reaction rate of carbon dioxide, $\text{g}/\text{m}^2\text{s}$,
 a CO_2 -binding capacity of concrete, kgCO_2/m^3 and
 t time, s .

By combining the above Equations 1 and 2 ($J = J_R$) and integrating over time ($X_C = 0$ when $t = 0$), the following solution is obtained:

$$X_{\text{carb}} = \sqrt{\frac{2D \cdot \Delta C_s \cdot t}{a}} \quad (3)$$

From Equation 3 it is seen that at constant conditions the depth of carbonation is proportional to the square root of time. If ΔC_s is constant in Equation 3, the equation can be presented in a more simplified form as:

$$X_{\text{carb}} = k_{\text{carb}} \sqrt{t} \quad (4)$$

where

- k_{carb} is coefficient of carbonation, $\text{mm}/\text{a}^{0.5}$.

The formula of carbonation (Equations 3 or 4) is an example of a simple analytic degradation model, which can be assumed to apply in constant conditions. It can be used in studying various material related effects on the rate of carbonation by laboratory tests. Also the effects of temperature, relative humidity and CO_2 content can be studied by laboratory tests.

The coefficient of carbonation depends e.g. on the permeability of concrete, quality of cement, possible cement replacements (blast furnace slag, silica fume etc.) and the environmental conditions. A less permeable concrete will have a slower carbonation rate. In wet concrete, carbonation is much slower than in only slightly moist concrete.

Another chemical phenomenon in the concrete that can initiate corrosion of reinforcement is **chloride penetration**. Chlorides in sufficiently rich concentrations can break the passive film around the reinforcement, even in very alkaline solutions.

The chloride penetration into concrete can be assumed to obey Fick's second law of diffusion. Accordingly in a semi-infinite wall the following solution can be derived for chloride content in concrete:

$$\frac{C - C_0}{C_s - C_0} = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D_{app} \cdot t}}\right) \quad (5)$$

where

C_0 is the initial chloride content of concrete pore water, mol/m³, and
 C_s chloride content at the surface of concrete, mol/m³,
 x distance from surface, m,
 D_{app} apparent diffusion coefficient, m²/s, and
 t time, s.

According to Bazant the error-function solution (Equation 5) can be closely approximated by the following parabola function [1]:

$$C = C_s \cdot \left(1 - \frac{X_{Cl}}{2\sqrt{3 \cdot D_{app} \cdot t}}\right)^2 \quad (6)$$

When Equation 6 is solved for X_{Cl} and the chloride content C is replaced by the critical chloride content C_{crit} the following function is obtained for the depth of critical chloride content:

$$X_{Cl} = \left(1 - \sqrt{\frac{C_{crit}}{C_s}}\right) 2\sqrt{3 \cdot D_{app} \cdot t} \quad (7)$$

where

X_{Cl} is depth of critical chloride content at moment t , m.

In this form it is obvious that the depth of the critical chloride content approximately obeys the "square-root-of-time"-law in the same way as the depth of carbonation:

$$X_{Cl} = k_{cl*} \sqrt{t} \quad (8)$$

where

$$k_{cl*} = 2\sqrt{3D_{eff}} \cdot \left(1 - \sqrt{\frac{C_{crit}}{C_s}}\right) \quad (9)$$

where

k_{cl*} coefficient of chloride penetration, mm/ $\sqrt{\text{year}}$ and

t time, year.

The Bazant-simplification offers a mathematically easy way to treat the problem of chloride ingress. The coefficient of chloride penetration depends on the type of cement.

From Equations 4 and 8 follows that in both cases the following differential equation can be applied:

$$\Delta X_c \approx \frac{k_c^2}{X_c} \cdot \Delta t \quad (10)$$

where

X_c is depassivative depth (either carbonation depth or the depth of critical chloride content), mm, and

k_c coefficient of depassivation (either carbonation or the chloride penetration), mm/a^{0.5}, and
t time, year.

At cracks the following theoretical dependency between the depassivative depth at a crack and the depassivative depth at uncracked areas is assumed [6]:

$$y(t) = \sqrt{2 \cdot \frac{D_R}{D_B} \cdot w \cdot x(t)} \quad (11)$$

where

y is depassivative depth at crack, mm,

x depassivative depth at uncracked surface, mm,

D_R diffusion coefficient at crack,

D_B diffusion coefficient at uncracked surface, and

w width of crack, mm.

According to Equation 11 the depassivative depth at a crack is proportional to the square root of the depassivative depth at an uncracked concrete surface and to the square root of the width of the crack.

2.2.2 Corrosion of Reinforcement

In aerial conditions corrosion can start only as a result of physical or chemical changes in concrete around the reinforcement. Physical changes are cracking and deterioration of concrete. As a result of cracking and deterioration a part of the steel is exposed and left without any physical or chemical protection. Chemical changes in concrete are carbonation and penetration of chlorides. Corrosion starts when either the front of carbonation or the critical chloride content reaches the reinforcement.

The process of corrosion is electrochemical. This means that electrons together with chemical compounds take part in the reactions. Two reactions are necessary to build up an electrochemical reaction: (1) anodic reaction, which generates electrons, and (2) cathodic reaction, which

consumes electrons. In concrete the anode and cathode areas of the reinforcement form an electric circuit with concrete as the electrolyte.

In corrosion reactions oxygen together with iron ions are chemically bound to ferrous or ferric hydroxides. The volume of corrosion products is much greater than the volume of the original metal. As a result of the expanding layer of corrosion products, pressure is produced in concrete at the anodic areas of rusting steel. If corrosion is allowed to continue the pressure will grow so great that it exceeds the tensile strength of concrete. As a result the concrete cover cracks. With continued corrosion and an increasing layer of corrosion products, concrete will spall from the surface of the structure. During this phase concrete cover completely loses its ability to protect the steel. In addition, the capacity of the structure may become questionable as a result of the reduced bond between steel and concrete. Furthermore the aesthetic appearance suffers from cracking and corrosion stains. So for many reasons the phases of corrosion cracking and spalling can be regarded as the limit states for service life of a concrete structure.

Applying Faraday's law, the corrosion rate can be used from direct electrical measurement:

$$r_{corr} = 11.6 \cdot i_{corr} \quad (12)$$

where

r_{corr} is corrosion rate, mm/yr, and
 i_{corr} mean value of the corrosion current, mA/cm².

The rate of corrosion in aerial conditions slows down with time. The reduction of corrosion rate is a result of corrosion products formed around the steel. Corrosion products seemingly act as extra electric resistance. The depth of corrosion can normally be presented in the form:

$$s_{corr} = k \cdot t^n \quad (13)$$

where

s_{corr} is depth of corrosion, μm ,
 t time, and
 k and n constants.

However, if the rate of corrosion is controlled by oxygen diffusion, as is the case for wet concrete, the rate of corrosion is assumed to be proportional with time:

$$s_{corr} = r_{corr} \cdot t \quad (14)$$

From Equations 13 and 14 follows that in outdoor conditions when the moisture content of concrete changes the exponent of time also changes.

For the maximum allowable depth of corrosion several limit states may be applied:

- $s_{max} = 0$
- $s_{max} = s_{crack}$ (limit state of cracking in concrete cover)
- $s_{max} = s_{capacity}$ (limit state based on the load bearing capacity of the structure).

In some cases, as in the case of prestressing steel, no corrosion can be allowed ($s_{\max} = 0$). It means that there is no active corrosion time allowed. Then the service life is determined only based on the depassivation time (t_0) which depends on the carbonation and chloride penetration processes. In general the service life can be determined as follows:

$$t_L = t_0 + t_1 \quad (15)$$

where

t_L is service life,
 t_0 depassivation time of corrosion, and
 t_1 active corrosion time (propagation time).

Normally the active corrosion time is defined using the limit state of cracking in concrete cover. In the Reference structure models the depth of corrosion corresponding to the limit state of cracking was evaluated by the formula:

$$s_{crack} = 100 \cdot \frac{c}{d} \quad (16)$$

where

s_{crack} is depth of crack, μm ,
 c thickness of concrete cover, m, and
 d diameter of steel rod, m.

There was a dependency assumed between the corrosion rates **at crack** and at uncracked areas. So, theories applied to general corrosion in concrete also apply to local corrosion at a crack. However, the rate of corrosion at a crack is also dependent on the width of the crack.

2.2.3 Frost Attack

Two types of frost action in concrete can be distinguished:

- internal damage and
- frost scaling of the surface.

Internal damage is caused by freezing of water inside concrete.

The vulnerability of concrete to internal frost attack can be evaluated in a laboratory using the theory of critical degree of saturation [3]. The degree of water saturation is defined as the relation of water filled volume of pores to the total volume of pores:

$$S = \frac{w_e}{V_{tot}} \quad (17)$$

where

S is degree of water saturation,
 w_e volume of evaporating water in concrete, and
 V_{tot} total volume of pores in concrete (air pores included).

According to the theory, the occurrence of frost failure depends on the degree of water saturation at the moment of freezing. The damage is always confined to such parts of the concrete where the degree of water saturation exceeds the critical value during freezing. The damage causes cracking, reduction in the strength and reduction in the E-modulus of concrete.

Normally the critical degree of saturation is exceeded only if all the capillary pores and a part of the smallest air pores have been filled with water [3]:

$$S_{crit} = \frac{V_{tot} - a_{crit}}{V_{tot}} = 1 - \frac{a_{crit}}{V_{tot}} \quad (18)$$

where

S_{crit} is critical degree of water saturation, and
 a_{crit} critical volume of (air filled) air pores.

There are theoretical methods by which it is possible to evaluate the critical volume of air pores if the air pore distribution of concrete is known. By this approach, the critical degree of saturation can be evaluated from Equation 18. The critical degree of saturation can also be determined by a direct freezing test [3].

To study the rate of water uptake into concrete a thin piece of concrete is placed on a grating so that it can continually absorb water through one of its surfaces. By weighing the specimen one can follow the increase of the water saturation which is nearly proportional to the square root of time. However there is a clear nick-point in the curve indicating the point where all the capillary pores are filled and only air pores, which have been capsulated by the capillary water are still air filled. The growth of the degree of water saturation can be mathematically modelled as:

$$S = A + B \cdot \log t \approx S_0 + B \cdot \log t \quad \text{or} \quad (19)$$

$$S = A' + C \cdot t^D \approx S_0 + C \cdot t^D$$

where A, A', B, C and D are parameters obtained from results of a capillary water saturation test [3]. Using Equation 19 it is possible to evaluate the time, which is needed to reach the critical degree of saturation.

In outdoor conditions the periods of moisture absorption and drying keep changing. The internal frost damage of concrete depends on the length of wet periods in relation to the time needed to reach the critical degree of saturation. So, the "failure" depends on both the outdoor conditions and the properties of concrete.

Surface scaling is caused by freezing on a concrete surface when it is in contact with a chloride solution. First the initial scaling occurs in cement paste while the aggregate grains are intact. Due to the gradually deeper and deeper scaling, coarser aggregate grains can also be lost.

The depth of scaling is related to the number of freeze-thaw cycles in the surface of concrete. The relationship between the freeze-thaw cycles in a frost-salt test and the depth of scaling can be evaluated by the Equation 20 [4]:

$$x_{sc} = k_{sc} \cdot N^k \quad (20)$$

where

- x_{sc} is scaling depth of concrete, m,
- N number of freeze-thaw cycles,
- k_{sc} coefficient of freeze-thaw scaling, m, and
- k acceleration factor.

The rate of scaling can be

- retarding ($k < 1$)
- even ($k = 0$)
- accelerating ($k > 1$).

An accelerating rate of scaling is usually a sign of poor frost resistance of concrete. A retarding scaling rate is a sign of inferior quality of concrete on the surface compared to the quality inside. In homogeneous concrete an even scaling rate is expected.

The rate of scaling in natural weather conditions cannot be evaluated exactly. However, rough evaluations can be made on the bases of laboratory test results. Assuming homogeneous concrete and a linear scaling rate, Equation 21 for the depth of scaling may be applied in natural conditions [4]:

$$x_{sc} = x_{sctest} \cdot \frac{\sum n_{eq}}{n_{test}} \quad (21)$$

where

- x_{sc} is depth of frost scaling in natural conditions, m,
- x_{sctest} depth of frost scaling in a laboratory test, m,
- n_{test} number of freeze-thaw cycles in a laboratory test, and
- $\sum n_{eq}$ equivalence number of freeze thaw cycles in natural conditions (corresponding to the cycles in a laboratory test).

2.3 Regional Models Created by Computer Simulation

The above-mentioned models for carbonation, chloride ingress and corrosion are applicable only in constant conditions. So they cannot be directly applied in natural conditions where the temperature and the moisture content of the structure always change. In freezing and thawing tests the conditions change but they usually differ much from natural conditions. So the results of a freeze-thaw test are not applicable as such to service life prediction.

The computer simulation method was used as the "missing link" between "laboratory models" and models for natural conditions, i.e. "regional models". The computer simulation process is comprised of the phases presented in Figure 2.

Based on the average compressive strength and air content of concrete, a default mix composition for concrete was determined, i.e. the proportions of cement, mixing water,

aggregate and air were determined. The parameters related to the heat and moisture transport in concrete and to the degradation were determined based on the default mix composition. The rate of carbonation and chloride penetration were calibrated so that the calculated depth of carbonation and the depth of critical chloride content would be equivalent to the average measured depths.

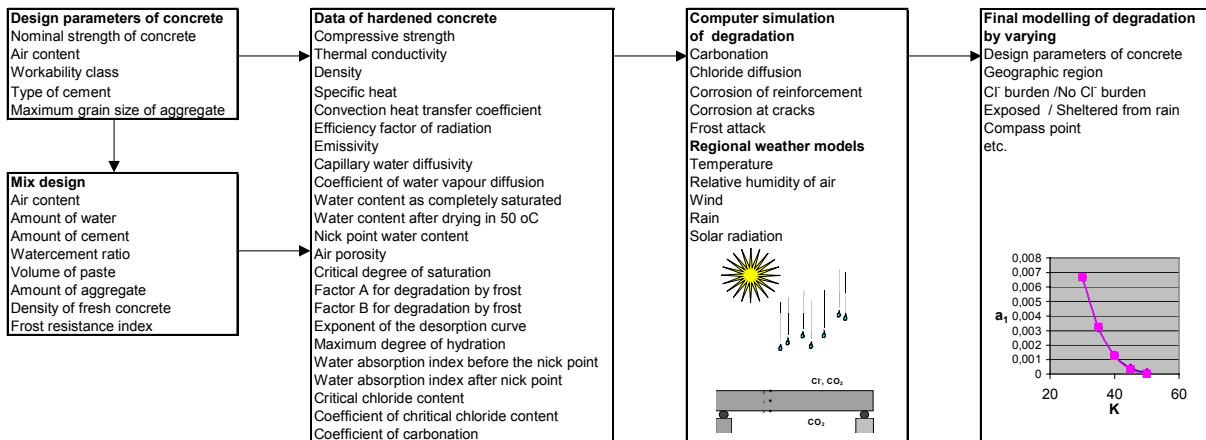


Figure 2. Scheme of the computer simulation method in producing degradation models.

In the computer simulation the temperatures, moisture contents and degradation processes of concrete structures are emulated as they occur in natural conditions. In the simulation program of VTT Building and Transport the weather models were based on the data gathered by the Finnish Meteorological Institute. They consist of data on temperature and relative humidity of the air as well as the velocity and direction of wind, amount of rain and intensity of solar radiation. Both daily and seasonal variations are considered in the weather models. The models of rain and solar radiation are statistical models which yield the correct amount of rain and solar radiation but the exact moments of rain and direct solar radiation are not given in a deterministic way.

A model structure, which in this case was a concrete slab or a wall, was burdened by the virtual weather. The outdoor weather conditions form the boundary conditions of a thermal and moisture mechanical problem to solve the temperature and moisture distributions in a structural cross-section. The incremental time was one hour. The total calculation may cover some months, some years or even some hundreds of years from the lifetime of the structure.

The possible increment of degradation both in concrete and in reinforcement was evaluated simultaneously with temperatures and moisture contents. Frost-salt scaling, internal frost damage, carbonation and penetration of chlorides in concrete and corrosion of reinforcement were followed, especially at the edge points of the structure. The degradation at cracks (carbonation, chloride penetration and corrosion) was evaluated separately.

The service life of a structure can be evaluated by the simulation method when the maximum allowable degradation has been defined. Service life is the period of time from the moment of manufacturing to the moment when the maximum allowable degradation is exceeded. The methods of the computer simulation are explained more completely elsewhere [7,8,9].

The degradation is evaluated as incremental degradation during each incremental time step. So the degradation models are applied in their differential forms. For instance the model of depassivation (ref. Equation 10) is used as:

$$X_{c;i} = \sum_i \Delta X_{c;i} = \sum_i k_{RH;i} \cdot k_{T;i} \cdot \frac{k_c^2}{X_{c;i-1}} \cdot \Delta t_i \quad (22)$$

where

$X_{c;i}$ is the depassivative depth, m,

$\Delta X_{c;i}$ incremental of the depassivative depth during the i^{th} time step, m,

$k_{RH;i}$ value of relative humidity factor for depassivation during the i^{th} time step,

$k_{T;i}$ value of the temperature factor of depassivation during the i^{th} time step, and

Δt_i i^{th} incremental time step, h.

The possible occurrence of internal frost attack can be determined by computer simulation using the theory of critical degree of water saturation. The theory can be easily attached to the calculation processes of the temperature and the moisture content of concrete which continuously follow the changes in outdoor conditions. The number of critical freezing events, i.e. the events when the critical degree of water saturation is exceeded simultaneously with freezing, is counted and the progress of damage as a function of critical freezing events is evaluated by degradation models.

The rate of scaling can be evaluated by counting the equivalent number of freeze-thaw-cycles and applying Equation 20.

The equations for corrosion rate are readily applicable for computer simulation. The total depth of corrosion is counted by integrating the incremental depths of corrosion produced during each time step.

The interactions of degradation factors can be determined during the calculation. For instance the depth of depassivation (both depth of carbonation and depth of critical chloride content) is dependent on the internal frost attack and the surface scaling. So instead of Equation 22 the true depth of depassivation can be evaluated from Equation 23:

$$X_{c;i} = \sum_i \Delta X_{c;i} = \sum_i k_{RH;i} \cdot k_{T;i} \cdot \frac{(k_D(D_i) \cdot k_{Carb})^2}{2 \cdot (X_{c;i-1} - X_{ssc;i-1})} \cdot \Delta t_i \quad (23)$$

where

$k_D(D)$ is the reduction factor for the coefficient of depassivation (may be different for carbonation and the chloride penetration),

D_i degree of damage in internal frost attack during the i^{th} time step, and

$X_{ssc;i}$ depth of surface scaling during the i^{th} time step, m.

Based on the results of computer simulation the degradation models were reformulated. The original "square-root of time"-rules for the carbonation or chloride ingress are no longer valid as

the varying moisture content and temperature and the possible interaction with internal frost damage and/or surface scaling changes the form of the degradation curve.

2.4 Local Models by Interpolation

As the amount of components in structures is usually great, it is not practical to create separate degradation models for each component. A simplified but still reasonably sensitive system was used instead. "Moisture index" and "chloride index" are component-related environmental parameters which take into account both the local and micro climatic effects that cannot be considered by regional models. Application of these indices calls for interpolation between basic cases of regional models.

2.4.1 Environmental Effects on Local Level

The moisture and chloride burdens are of course dependent on the environment of the structure. The burdens are very different for bridges, tunnels, canals, dams, pole basements, quays, lighthouses etc.. In the following sections the moisture index and the chloride index are only considered for bridge structures.

To be able to specify the environmental burdens for a bridge structure it is important to know what is the obstacle crossed by the bridge: gulf of a sea, river, road or railroad. Another environmental factor affecting the chloride index is the amount of deicing salt spread on the over-passing and the under-passing roads. The amount of salt spread on the roads is defined in Finland by the winter maintenance class.

The anti-skid treatment in Finland for public roads is presented in Table 1. The amount of salt is given as tons per road kilometer.

Table 1. The anti-skid treatment in Finland for public roads.

Winter maintenance class	Basis for Classification	Anti-skid treatment	Evaluated amount of salt t/km/year
Isk	2-track super highways ADT > 12000	always de-icing salted	20
Is	1-track super highways ADT 6000-12000	always de-icing salted	7-14
I	ADT 3000...6000	always de-icing salted	5-10
Ib	ADT 1500...3000	always de-icing salted excluding Lapland	2-4
II	ADT 350...1500	sometimes de-icing salted in Southern-Finland in autumn	-
III	ADT < 350	contractor may have spread salt	-
K1	Roads for bicycle and pedestrian traffic	no anti-skid treatment	-
K2	Lightly trafficked roads	no anti-skid treatment	-

	for bicycle and pedestrian traffic		
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2.4.2 Component Level Environmental Effects

Many factors affect the salt burden and the moisture burden for different structural parts of bridges. Not only is the total amount of salt spread on the over-passing and the under-passing roads important, but also the location of the structure in relation to the roads. The real moisture index and the chloride index are highly dependent on the distance of the structure from the over-passing and the under-passing roads and the sheltering effects of other structures.

The environmental burdens at the micro level were evaluated as the degree of exposure both with respect to moisture and chlorides. As for chlorides the degree of exposure was defined both with respect to the over-passing and the under-passing road. The final chloride index of a structural part is obtained as the sum of the chloride burdens from the over-passing and under-passing roads, both being determined as the product of the degree of the chloride exposure of the structural part and the relative de-icing burden of the road.

The moisture index and the chloride index were used as interpolation parameters. When the degradation curves for the maximal and minimal stresses for both moisture and chlorides (at chloride index 0 and 1 and at moisture index 0 and 1) have been determined the degradation models for intermediate stresses can be determined by interpolation.

2.5 Calibration of Models by Field Tests

The validity of the models was examined by field tests carried out on "reference bridges". The network of reference bridges was established in the early 1990's for studying the real degradation on bridges. The reference bridges were selected based on building material, bridge type and the year of construction as a sample to represent all the bridges on public roads in Finland. In 1997 all 120 reference bridges were inspected at least once.

During inspection, measurements of concrete cover were performed at the bridge site. Also concrete samples were drilled from the bridges for laboratory tests. The laboratory tests were comprised of determining compressive strength, rate of capillary water uptake, porosity, depth of carbonation and chloride penetration.

A statistical analysis on all the field and laboratory research results was carried out in 1998. As a result statistical data on compressive strength, porosity of concrete, rate of capillary water uptake, rate of carbonation, rate of chloride ingress and thickness of concrete cover in concrete bridges of Finnish public roads was obtained.

From the degradation data gathered from the reference bridge studies, the models of carbonation and chloride penetration could be calibrated. Statistical data of these degradation phenomena in various parts of bridges was also obtained.

3 MODEL FUNCTIONS

The full formulae of the Reference structure models are presented in the APPENDIX. Only the mathematical form and some rules for application are presented below.

3.1 Model Functions of Degradation at Regional (Macro) Level

Based on the results obtained from computer simulation, the degradation in both the depassivation phase and the active corrosion phase was given a simple model function of the following form:

Depassivation:

$$f_0 = a_0 \cdot t^{n_0} \quad (24)$$

where

f_0 is penetration of carbonation or critical chloride content in relation to the concrete cover,
 t time from the manufacture of the structure, year,
 a_0 coefficient, and
 n_0 exponent of time.

Active corrosion:

$$f_1 = a_1 \cdot t^{n_1} \quad (25)$$

where

f_1 is depth of corrosion of reinforcing steel in relation to the critical depth of corrosion causing cracking of concrete cover,
 t time from depassivation, year,
 a_1 coefficient, and
 n_1 exponent of time.

In the depassivation phase the function f_0 starts from 0 at the beginning of service life and attains the value 1 at the moment of depassivation (when carbonation or the critical chloride content reaches the depth of reinforcement). In the active corrosion phase the function f_1 starts at 0 immediately after depassivation and attains the value 1 when the concrete cover cracks as a result of corrosion.

In the models of surface damage the coefficient a and the exponent n (for both depassivation and active corrosion phase) were assumed to be dependent on several environmental, structural and material parameters:

Surface damage

Depassivation:

$$a_0 = k_{GIS} \cdot k_{H/V} \cdot k_{Cem} \cdot k_{Cov} \cdot k_{Frost} \cdot F_{a0}(K) \quad (26 a)$$

$$n_0 = F_{n_0}(K) \quad (26 \text{ b})$$

Active corrosion:

$$a_1 = k_{GIS} \cdot k_{Cov} \cdot k_{Frost} \cdot F_{a_1}(K) \quad (27 \text{ a})$$

$$n_1 = F_{n_1}(K) \quad (27 \text{ b})$$

where

k_{GIS} is Regional environmental factor. GIS (= Geographic Information System) refers to climatic data on regional level (temperature, relative humidity, rain, wind, solar radiation),

$k_{H/V}$ Surface direction factor (Horizontal/ Vertical),

k_{Cem} Cement factor,

k_{Cov} Cover thickness factor,

k_{Frost} Frost attack factor, and

$F(K)$ Function depending on the nominal compressive strength of concrete.

The factors for a_1 and a_0 may not be the same in spite of the same denotation.

The climatic data was obtained from weather statistics gathered by the Finnish Meteorological Institute [2]. The climatic data were used as statistical models in computer simulation. In the Finnish models three different cities were selected to represent climatic conditions in Finland: Helsinki (Southern Finland), Jyväskylä (Middle Finland) and Sodankylä (Northern Finland).

Similar to the surface damage models, the models for crack damage are also defined by Equations 24 and 25. However the coefficient a and the exponent n are different:

Crack damage

Depassivation

$$a_0 = k_{Cw}^{\frac{1}{2}} \cdot k_{H/V}^{\frac{1}{2}} \cdot k_{GIS}^{\frac{1}{2}} \cdot k_{Cem}^{\frac{1}{2}} \cdot k_{Cov}^{\frac{1}{2}} \cdot k_{Frost}^{\frac{1}{2}} \cdot F_{a_0;c}(K)^{\frac{1}{2}} \quad (28 \text{ a})$$

$$n_0 = F_{n_0;c}(K) \quad (28 \text{ b})$$

Active corrosion

$$a_1 = k_{Cw} \cdot k_D \cdot k_{GIS} \cdot k_{Cov} \cdot F_{a_1;c}(K) \quad (29 \text{ a})$$

$$n_1 = F_{n_1;c}(K) \quad (29 \text{ b})$$

where

k_{GIS} is Regional environmental factor. GIS (= Geographic Information System) refers to climatic data on regional level (temperature, relative humidity, rain, wind, solar radiation),

$k_{H/V}$ Surface direction factor (Horizontal/ Vertical),

k_{Cem} Cement factor,

k_{Cov} Cover thickness factor,

k_{Frost} Frost attack factor, and

$F(K)_c$ Function depending on the nominal compressive strength of concrete.

Frost attack only

For the cases when it is important to know the rate of degradation based only on frost attack, the following models were created. Although the mechanism of internal frost attack and salt-frost attack is different the appearance of the basic formula is the same.

$$f = a \cdot t^n \quad (30)$$

where

f is degradation in relation to the maximum allowable degradation ($f = 0 \dots 1$),

t time, year

a coefficient, and

n exponent.

In the case of salt-frost attack, the maximum allowable degradation was defined as the state when some of the coarse aggregate particles are loosened as a result of frost scaling. The depth of disintegration of concrete is 0 - 20 mm. For the case of internal frost damage, the maximum allowable degradation is defined as the state when the integrity of concrete is seriously reduced. This reduction means cracking and minor spalling are visible and the dynamic modulus of elasticity of concrete is reduced to more than 2/3 of the original.

The coefficient a is determined from the following

$$a = k_{GIS} \cdot k_{H/V} \cdot k_{Cem} \cdot F_a(K; A) \quad (31 a)$$

where

A is the air content, %.

The degradation is assumed to be linear. So the value of exponent is:

$$n = 1 \quad (31 b)$$

Conversion for water-to-cement ratio

Instead of nominal strength of concrete, the water-to-cement ratio may be used as a parameter of degradation models. The following conversion rule may be applied between the nominal strength and water-to-cement ratio for cements CEM 42.5. This rule was also used in the development phase of the degradation models:

$$w/c = \frac{28.8}{21.5 + K} \quad (32)$$

where

w/c is water-to-cement ratio, and

K nominal strength of concrete, MPa,

The Equation 32 is valid in the range 20 MPa K 45 MPa. The relationship between the nominal strength and the average strength is given as:

$$K = f_{cm} - 8 \quad (33)$$

where

f_{cm} is average compressive strength of concrete, MPa.

3.2 Interpolation Process for Local Models

The quantities a_0 , n_0 , a_1 and n_1 are first determined for four basic cases of moisture and chloride burden:

1. moisture index 0/ chloride index 0.
2. moisture index 1/ chloride index 0
3. moisture index 0/ chloride index 1 and
4. moisture index 1/ chloride index 1.

The chloride index 1 corresponds to the amount of chlorides received by unsheltered concrete on top of a bridge deck with the highest winter maintenance class. The chloride index 0 represents the case when no chloride is spread on bridges (depassivation is possible only by carbonation). The moisture index 1 corresponds to the amount of external water (rain, melting, splash) received by unsheltered concrete on top of a bridge deck and moisture index 0 to the amount of external water under a bridge deck provided with a water membrane.

The degradation curves are then refined by interpolation based on both moisture index m (between 0-1) and chloride index c (between 0-1). The moisture index m and the chloride index c are dependent on environmental burdens at both a meso (local) level and at a micro level.

The interpolation is first performed with respect to moisture index m for both cases $c=1$ and $c=0$ as follows. .

$$a(m) = a(0) + m(a(1) - a(0)) \quad (34 a)$$

$$n(m) = n(0) + m(n(1) - n(0)) \quad (34 b)$$

Then the final interpolation is performed with respect to chloride index c as follows:

$$a(m,c) = a(m,0) + c(a(m,1) - a(m,0)) \quad (35 a)$$

$$n(m,c) = n(m,0) + c(n(m,1) - n(m,0)) \quad (35 b)$$

The method is the same for surface damage and crack damage. For more details, see the full description of the models in the APPENDIX.

4 APPLICATION OF THE MODELS IN LIFECON LMS

4.1 Properties and Use of Reference Structure Models

The Reference structure models are in conformity with the requirements of models presented in the Handbook of LIFECON Life Cycle Management System (ref. Deliverable D1.1). Actually the Reference structure models have been used in many examples of both the Handbook and in this Deliverable (ref. Part 1: "Markov Chain based LCC Analysis") for illustrating the processes of LIFECON LMS.

The following input data need to be specified for the models:

Surface damage:

1. Geographical location (1= Coastal Finland, 2= Middle Finland, 3= Northern Finland)
2. Obstacle to cross over (1= road or railway, 2= sea, 3= other water)
3. Winter maintenance class
 - 3.a. for a road over-passing a bridge
 - 3.b. for a road under-passing a bridge
4. Structure (index number from the Table of structural parts)
5. Nominal strength of concrete (MPa)
6. Cement type
7. Air content of concrete (%)
8. Thickness of concrete cover (mm)

The Table of structural parts is provided with default values (horizontal/ vertical surface, moisture index and chloride index), but the user can change these values if necessary.

The same above mentioned input data is given also for crack damage. In addition the width of the crack (mm) and the diameter of the main reinforcement steel (mm) are given.

In LIFECON LMS the computer programs for optimization and planning are linked to the database of the system. It is assumed that the above presented parameter data are obtained directly from the database. If some data is lacking default values are used instead. So, in automatic processes of the management system no data is inserted manually. However, in manual processes the designer can specify all data.

4.2 Restrictions of Models

The following are the main restrictions for the use of the model:

- The degradation of structures under water, in the splash zone or under the ground level has not been modelled.
- The special durability problems of prestressed concrete structures have not been considered in the models.
- The degradation models presented here do not take into account the possible leakage of other structures that shelter them.

5 REFERENCES

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APPENDIX: Reference structure models

Degradation Models for Surface Damage and Corrosion at Cracks

Depassivation:

$$f_0 = a_0 \cdot t^{n_0} \quad (1)$$

where

f_0 is depth of carbonation (no chlorides) or critical chloride content in relation to the concrete cover,

t time from the manufacture of the structure,

a_0 coefficient, and

n_0 exponent of time.

Active corrosion:

$$f_1 = a_1 \cdot t^{n_1} \quad (2)$$

where

f_1 is depth of corrosion of reinforcing steel in relation to the critical depth of corrosion causing cracking of concrete cover,

t time from depassivation,

a_1 coefficient, and

n_1 exponent of time.

Interpolation of coefficients a and exponents n

With respect to moisture burden

$$a(m,0) = a(0,0) + m(a(1,0) - a(0,0)) \quad (3 a)$$

$$a(m,1) = a(0,1) + m(a(1,1) - a(0,1)) \quad (3 b)$$

$$n(m,0) = n(0,0) + m(n(1,0) - n(0,0)) \quad (3 c)$$

$$n(m,1) = n(0,1) + m(n(1,1) - n(0,1)) \quad (3 d)$$

m is moisture index,

$a(m,0)$ constant coefficient at moisture burden m and at chloride burden 0,

$a(m,1)$ constant coefficient at moisture burden m and at chloride burden 1,

$n(m,0)$ exponent of time at moisture burden m and chloride burden 0, and

$n(m,1)$ exponent of time at moisture burden m and chloride burden 1.

With respect to the chloride burden

$$a(m,c) = a(m,0) + c(a(m,1) - a(m,0)) \quad (4 a)$$

$$n(m,c) = n(m,0) + c(n(m,1) - n(m,0)) \quad (4 b)$$

c is chloride index,

$a(m,c)$ constant coefficient at moisture burden m and chloride burden c, and

$n(m,c)$ exponent of time at moisture burden m and chloride burden c.

Moisture index m

$$m = \text{Max}(\text{object related default value}; \text{component related default value}) \quad (5)$$

Table 1. Degree of exposure with respect to moisture. Object related default values for bridges.

Obstacle to cross over	m
Road	0
Railroad	0
Sea	0.4
Other water	0.2

Table 2. Degree of exposure with respect to moisture. Component related default values for bridges.

Component	m
Side wall	0.7
Frontal wall	0.1
Wing wall	0.8
Bearing pad	0.1
Column at the edge	0.1
Column in the middle	1
Abutment at the edge	0.8
Abutment in the middle	0.5
Edge beam	0.8
Side of the deck	0.5
Deck without water membrane	1
Deck with water membrane	1
Deck, underside	0
Girder, beam	0
Girder, arch	0
Girder, box beam, under side	0
Secondary girder	0
Pylon	0.9

Chloride index c

The final chloride index of a structure is obtained as the sum of the chloride burdens from the over-passing and under-passing roads, each determined as the product of the degree of the chloride exposure of the structure and the level of the deicing burden of the road.

$$c = \text{degree of exposure of component with respect to road} * \text{deicing burden of the road} + \text{degree of exposure of component with respect to crossing road} * \text{deicing burden of crossing road} \quad (6)$$

Table 3. Relative deicing burden of the road or the crossing road depending on the winter maintenance class

Winter maintenance class	Deicing burden
Isk	1
Is	0.55
I	0.38
Ib	0.15
Lower	0

Table 4. Degree of exposure with respect to chlorides. Component related default values for bridges.

Component	Degree of expose	
	for road	for crossing road
Side wall	0.2	0.4
Frontal wall	0	0.5
Wing wall	0.3	0.4
Bearing pad	0	0.4
Column at the edge	0	0.4
Column in the middle	1	0
Abutment at the edge	0.1	0.9
Abutment in the middle	0	0.6
Edge beam	0.1	0.9
Side of the deck	0	0.6
Deck without water membrane	1	0
Deck with water membrane	1	0
Deck, underside	0	0.2
Girder, beam	0	0.2
Girder, arch	0	0.2
Girder, box beam, under side	0	0.2
Secondary girder	0	0.2
Pylon	1	0

Coefficients for Surface Damage

Chloride burden 0 / Moisture burden 0

Parameters:

K nominal strength of concrete, MPa
C thickness of concrete cover, mm

$$a_0 = k_{Cov} \cdot k_{Cem} \cdot \left(-0.012 + \frac{5.74}{K} \right) \quad (7)$$

$$k_{Cov} = \frac{10}{\text{Min}(70; C)} \quad (8)$$

Table 5. Coefficient of cement type for carbonation

Cement	k_{Cem}
CEM I A	1.00
CEM II/A-S	1.14
CEM II/B-S	1.25
CEM II/A-D	1.19
CEM II/A-LL	1.00
CEM II/A-M	1.00
CEM III/A	1.46
CEM III/B	1.56

$$n_0 = 0.409 + \frac{5.02}{K} \quad (9)$$

$$a_1 = k_{GIS} \cdot k_{Cov} \cdot \left(-0.01 + \frac{0.93}{K} \right) \quad (10)$$

Table 6. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.86
Northern Finland	0.62

k_{Cov} see Equation 8.

$$n_1 = 0.407 + 0.011 \cdot K \quad (11)$$

Coefficients for Surface Damage

Chloride burden 0 / Moisture burden 1

Parameters:

K	nominal strength of concrete, MPa
C	thickness of concrete cover, mm
A	air content of concrete, %

$$a_0 = k_{GIS} \cdot k_{H/V} \cdot k_{Cem} \cdot k_{Cov} \cdot k_{frost} \cdot \left(-0.03 + \frac{2.84}{K} \right) \quad (12)$$

Table 7. Regional coefficient for carbonation.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.9
Northern Finland	0.76

Table 8. Coefficient of surface direction

Surface direction	$k_{H/V}$
Horizontal	1
Vertical	0.93

k_{Cem} see Table 5.

k_{Cov} see Equation 8.

$$k_{Frost} = \text{Max} \left(0.6; \frac{132}{27 + 1.48 \cdot A^{3.4}} \right) \cdot \text{Max} \left(0.5; \left(\frac{85.6 - 0.77 \cdot K}{21.5 + K} \right)^3 \right) \quad (13)$$

$$n_0 = 0.71 - \frac{13.2}{K^{1.5}} \quad (14)$$

$$a_1 = k_{GIS} \cdot k_{Cov} \cdot k_{Frost} \cdot 0.075 \quad (15)$$

Table 9. Regional coefficient for corrosion

Region	k_{GIS}
Coastal Finland	1
Middle Finland	1
Northern Finland	0.76

k_{Cov} see Formula 8.

k_{Frost} see Formula 13

$$n_1 = 1.18 \quad (16)$$

Coefficients for Surface Damage

Chloride burden 1 / Moisture burden 0

Parameters:

K	nominal strength of concrete, MPa
C	thickness of concrete cover, mm
A	air content of concrete, %

$$a_0 = 0.5 \cdot k_{Cov} \cdot k_{Cem} \cdot \left(-0.432 + \frac{27.7}{K} \right) \quad (17)$$

k_{Cov} see Equation 8.

Table 10. Coefficient of cement type for chloride penetration

Cement	k_{Cem}
CEM I A	1.00
CEM II/A-S	0.83
CEM II/B-S	0.71
CEM II/A-D	0.23
CEM II/A-LL	1.00
CEM II/A-M	1.00
CEM III/A	0.43
CEM III/B	0.27

$$n_0 = 0.413 + \frac{85800}{K^{3.5}} \quad (18)$$

$$a_1 = k_{GIS} \cdot k_{Cov} \cdot \left(-0.15 + \frac{8.40}{K} \right) \quad (19)$$

Table 11. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.84
Northern Finland	0.55

k_{Cov} see Equation 8.

$$n_1 = 1.06 - \frac{65.9}{K^{1.5}} \quad (20)$$

Coefficients for Surface Damage

Chloride burden 1 / Moisture burden 1

Parameters:

K	nominal strength of concrete, MPa
C	thickness of concrete cover, mm
A	air content of concrete, %

$$a_0 = k_{GIS} \cdot k_{H/V} \cdot k_{Cem} \cdot k_{Cov} \cdot k_{Frost} \left(-0.01 + 3.88 \cdot 10^{-6} \cdot K^3 \right) \quad (21)$$

Table 12. Regional coefficient.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.78
Northern Finland	0.71

Table 13. Coefficient of surface direction.

Surface direction	$k_{H/V}$
Horisontal	1
Vertical	0.93

$$k_{Cov} = \frac{35}{\text{Min}(70; C)} \quad (22)$$

$$k_{Frost} = 0.8 * \left(10 \cdot \frac{\left(\frac{28.8}{21.5 + K} \right)^{1.25}}{A^{0.5}} - 1 \right) \quad (23)$$

$$n_0 = 0.976 - 1.3 \cdot 10^{-5} \cdot (55 - \text{Min}(50; K))^3 \quad (24)$$

$$a_1 = k_{GIS} \cdot k_{Cov} \cdot k_{Frost} \cdot 0.3 \quad (25)$$

Table 14. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.93
Northern Finland	0.71

k_{Frost} see Formula 23.

$$n_1 = 1 \quad (26)$$

Coefficients for Crack Damage

Chloride burden 0 / Moisture burden 0

Parameters:

- K nominal strength of concrete, MPa
 C thickness of concrete cover, mm
 W width of crack, mm
 D diameter of the corroding steel bar at crack, mm

$$a_0 = 4.75 \cdot k_{Cw}^{\frac{1}{2}} \cdot k_{Cem}^{\frac{1}{2}} \cdot k_{Cov}^{\frac{1}{2}} \cdot \left(-0.012 + \frac{5.74}{K} \right)^{\frac{1}{2}} \quad (27)$$

$$k_{Cw} = \text{Min} \left(1; \frac{W}{0.2} \right) \quad (28)$$

k_{Cov} see Equation 8.

k_{Cem} see Table 5.

$$n_0 = 0.5 \cdot \left(0.409 + \frac{5.02}{K} \right) \quad (29)$$

$$a_1 = 0.11 \cdot k_{Cw} \cdot k_D \cdot k_{GIS} \cdot k_{Cov} \cdot \left(-0.01 + \frac{0.93}{K} \right) \quad (30)$$

$$k_{Cw} = 1 + \frac{1}{1 + \frac{1}{W}} \quad (31)$$

$$k_D = \frac{D}{20} \quad (32)$$

Table 15. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.86
Northern Finland	0.62

k_{Cov} see Equation 8.

Coefficients for Crack Damage

Chloride burden 0 / Moisture burden 1

Parameters:

- K nominal strength of concrete, MPa
 C thickness of concrete cover, mm
 A air content of concrete, %
 W width of crack, mm
 D diameter of the corroding steel bar at crack, mm

$$a_0 = 4.75 \cdot k_{Cw}^{\frac{1}{2}} \cdot k_{H/V}^{\frac{1}{2}} \cdot k_{GIS}^{\frac{1}{2}} \cdot k_{Cem}^{\frac{1}{2}} \cdot k_{Cov}^{\frac{1}{2}} \cdot k_{Frost}^{\frac{1}{2}} \cdot \left(-0.033 + \frac{2.85}{K} \right)^{\frac{1}{2}} \quad (33)$$

k_{Cw} see Equation 28.

Table 15. Regional coefficient for carbonation.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.9
Northern Finland	0.76

Table 16. Coefficient of surface direction.

Surface direction	$k_{H/V}$
Horisontal	1
Vertical	0.93

k_{Cem} see Table 5.

k_{Cov} see Equation 8.

k_{Frost} see Equation 13.

$$n_0 = 0.5 \cdot \left(0.71 - \frac{13.24}{K^{1.5}} \right) \quad (34)$$

$$a_1 = 0.11 \cdot k_{Cw} \cdot k_D \cdot k_{GIS} \cdot k_{Cov} \cdot 0.075 \quad (35)$$

k_{Cw} see Equation 31.

k_D see Equation 32.

k_{Cov} see Equation 8.

Table 17. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	1
Northern Finland	0.76

$$n_1 = 1.18 \quad (36)$$

Coefficients for Crack Damage

Chloride burden 1 / Moisture burden 0

Parameters:

- K nominal strength of concrete, MPa
 C thickness of concrete cover, mm
 A air content of concrete, %
 W width of crack, mm
 D diameter of the corroding steel bar at crack, mm

$$a_0 = 10 \cdot 0.5^{\frac{1}{2}} \cdot k_{Cw}^{\frac{1}{2}} \cdot k_{Cem}^{\frac{1}{2}} \cdot k_{Cov}^{\frac{1}{2}} \cdot k_{Frost}^{\frac{1}{2}} \cdot \left(-0.43 + \frac{27.7}{K} \right)^{\frac{1}{2}} \quad (37)$$

k_{Cw} see Equation 28.

k_{Cov} see Equation 8.

k_{Cem} see Table 10.

$$n_0 = 0.5 \cdot \left(0.413 + \frac{85800}{K^3} \right) \quad (38)$$

$$a_1 = 0.11 \cdot k_{Cw} \cdot k_D \cdot k_{GIS} \cdot k_{Cov} \cdot \left(-0.15 + \frac{8.40}{K} \right) \quad (39)$$

k_{Cw} see Equation 31.

k_D see Equation 32.

Table 18. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.84
Northern Finland	0.55

k_{Cov} see Equation 8.

$$n_1 = 1.06 - \frac{65.9}{K^{1.5}} \quad (40)$$

Coefficients for Crack Damage

Chloride burden 1 / Moisture burden 1

Parameters:

- K nominal strength of concrete, MPa
 C thickness of concrete cover, mm
 A air content of concrete, %
 W width of crack, mm
 D diameter of the corroding steel bar at crack, mm

$$a_0 = 10 \cdot k_{Cw}^{\frac{1}{2}} \cdot k_{H/V}^{\frac{1}{2}} \cdot k_{GIS}^{\frac{1}{2}} \cdot k_{Cem}^{\frac{1}{2}} \cdot k_{Cov}^{\frac{1}{2}} \cdot k_{Frost}^{\frac{1}{2}} \cdot \left(-0.01 + 3.88 \cdot 10^{-6} \cdot (55 - \text{Min}(50; K))^3 \right)^{\frac{1}{2}} \quad (41)$$

k_{Cw} see Equation 28.

k_{Cov} see Equation 22.

k_{Cem} see Table 10.

Table 19. Regional coefficient.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.78
Northern Finland	0.71

Table 20. Coefficient of surface direction.

Surface direction	$k_{H/V}$
Horizontal	1
Vertical	0.93

k_{Frost} see Equation 23.

$$n_0 = 0.5 \cdot \left(0.97 - 1.3 \cdot 10^{-5} \cdot (55 - \text{Min}(50; K))^3 \right) \quad (42)$$

$$a_1 = k_{GIS} \cdot k_{Cw} \cdot k_D \cdot k_{Cov} \cdot k_{Frost} \cdot 0.3 \cdot K \quad (43)$$

k_{Cw} see Equation 31.

k_D see Equation 32.

Table 14. Regional coefficient for corrosion.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.93
Northern Finland	0.71

k_{Cov} see Equation 22.

k_{Frost} see Equation 23.

$$n_1 = 1.0 \quad (44)$$

Coefficients for Frost Attack only

The degree of damage is evaluated using equation:

$$f = a \cdot t^n \quad (45)$$

where

- f is amount of damage in relation to the maximum allowable amount of damage,
 t time from the manufacture of the structure,
 a coefficient, and
 n exponent of time.

The interpolation method presented by Equations 3 and 4 is used.

For Chloride burden 0/ Moisture burden 0 and Chloride burden 1/ Moisture burden 0 the value of Equation 45 is 0.

Chloride burden 0 / Moisture burden 1

Parameters:

- K nominal strength of concrete, MPa
 A air content of concrete, %

$$a = 0.0069 \cdot k_{GIS} \cdot \text{Max}\left(0.6; \frac{132}{27 + 1.48 \cdot A^{3.4}}\right) \cdot \text{Max}\left(0.5; \left(\frac{85.6 - 0.77 \cdot K}{21.5 + K}\right)^3\right) \quad (46)$$

Table 15. Regional coefficient for frost attack.

Region	k_{GIS}
Coastal Finland	1
Middle Finland	0.95
Northern Finland	0.83

n =1

Chloride burden 1 / Moisture burden 1

Parameters:

- K nominal strength of concrete, MPa
 A air content of concrete, %

$$a = 0.0174 \cdot k_{GIS} \cdot \left(10 \cdot \frac{\left(\frac{28.8}{21.5 + K}\right)^{1.25}}{A^{0.5}} - 1\right) \quad (47)$$

k_{GIS} see Table 15.

n =1 (48)